Time Evolution of a Differentiated Oligopoly: The Case of Sustainable Wine*

Michèle Breton[†], HEC Montréal
Lucia Sbragia[‡], Durham University Business School

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Abstract

We study the time evolution of a vertically and horizontally differentiated oligopolistic industry, where firms compete in quantity and are divided into groups producing one variety of a substitutable product. We assume that firms can periodically revise their decision about which variety to produce. For a general oligopoly with two varieties, we characterize the industry composition in the steady state as a function of the parameter values. Our results are applied to the case of the sustainable wine industry.

Keywords: differentiated oligopoly, wine industry, sustainable production, dynamic market composition.

1 Introduction

The concept of "sustainability" (UN 2005) is gaining in popularity in many production processes (e.g. coffee, seafood, and fashion) and it has recently reached the wine industry. Even though there is no universal definition of sustainable wine, embracing a sustainability principle in the wine-making industry generally involves environmentally friendly business practices, socially responsible use of human and community resources, and long-term economic viability.[1] In general, sustainable conduct applies to every aspect of wine production,

*Research supported by NSERC (Canada).
†HEC Montréal, Department of Decision Sciences. Email: michele.breton@hec.ca.
‡Durham University Business School, Department of Economics. Email: lucia.sbragia@durham.ac.uk.
1 See, for instance, the Global Wine Sector Environmental Sustainability Principles described in the GWS-ESP brochure, available at https://www.sustainablewinegrowing.org/docs/cswa_gwsesp_brochure.pdf

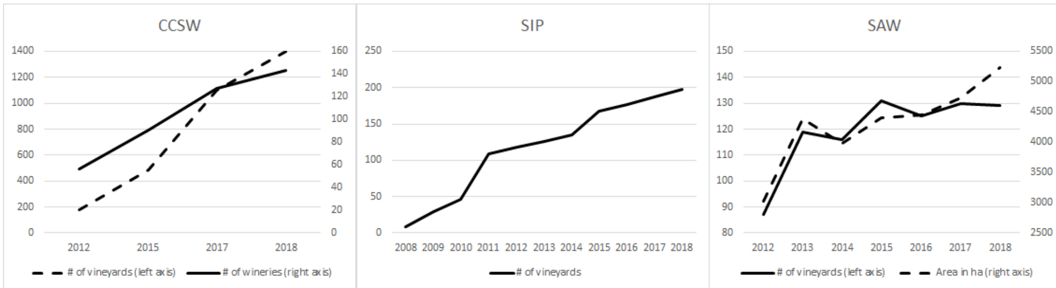

Figure 1: Evolution of the membership of sustainable wine certification programs. Left panel: Certified California Sustainable Winegrowing (CCSW) program. Center panel: Sustainable In Practice (SIP) certification program. Right panel: Sustainable Australia Winegrowing (SAW) program.

including the vineyard, winery, surrounding habitat and ecosystem, employees, and community.

To illustrate the evolution of sustainable wine production around the world, Figure 1 shows a steady increase in the number of vineyards, wineries, and farmed areas reported by three certification programs in the U.S. and Australia.

The Certified California Sustainable Winegrowing (CCSW) program was launched in 2010; by 2018, 70% of wine produced and 25% of total farmed acreage in California was CCSW certified. The Sustainable In Practice (SIP) certification program was initiated in 2008. At that time 3,700 acres were certified; today the number of certified acres in California and Michigan is over 41,100 and the program shows a retention rate of 90%. The Sustainable Australia Winegrowing program, open to grape growers only, started in 2011; in 2018, it accounted for 5,239 hectares of farming area. The Sustainable Winegrowing New Zealand (SWNZ) program reports that, in 2016, 98% of New Zealand's vineyard production area was SWNZ certified. "Old Word" wine producing countries are also taking steps in this innovative industry. For instance, in 2011, the Italian Ministry for the Environment, Land and Sea launched a national program named VIVA. The VIVA label is released to wine-producing companies that meet the required sustainability standards; to date, the VIVA certification has been granted to 22 products commercialized by 11 firms.

The main characteristics of the sustainable wine industry can be summarized as follows:

i) Coexistence of sustainable and conventional wine production;

ii) Possibility for grape growers and wine producers to review their production practices;

iii) Presence of certification costs;

iv) Presence of knowledge spillovers among certified sustainable wine producers;

v) Premium associated with having a sustainability certification label.

The aim of this paper is to build and study a general differentiated oligopoly model able to incorporate these specific features. More specifically, our model extends the duopoly model proposed in Singh & Vives (1984) in three different ways.[2] Firstly, in our model, each variety of the product is produced by a group of homogeneous firms that compete in quantity. This is different, and more general, than having N firms producing one variety each like in Vives (1985), Häckner (2000), or Amir & Jin (2001), as it accounts not only for interbrand competition, that is, competition between differentiated products, but also for competition between homogeneous products. Furthermore, our model differs from Singh & Vives (1984) in that each firm pays a fixed cost related to the product variety supplied. Finally, for a given industry size, we allow firms to periodically review their decision about the variety they are producing, which makes the composition of the oligopoly dynamic. This is captured by allowing firms to decide on which production technology to adopt, based on the different groups's relative economic performance. This behavior can be represented, for instance, through standard replicator dynamics. We assume that industry members are myopic with respect to the evolution of the industry composition over time, which is quite common in the literature pertaining to the exploitation of natural resources (see, e.g., Sethi & Somanathan 1996; Noailly, van den Bergh & Withagen 2003; Bischi, Lamantia & Sbragia 2004; Bischi, Lamantia & Radi 2015; and Petrohilos-Andrianos & Xepapadeas 2017).[3]

In the first part of the paper, we solve the general differentiated oligopoly model for the static Cournot solution corresponding to a given industry size and composition, in the case where there are two product varieties (e.g. green and brown). The sensitivity analysis of the equilibrium solution to the model parameters generalizes part of the investigation carried out in Theilen (2012), Kopel et al. (2017), and Dou & Ye (2017). In this analysis, the impact of intensifying intra-group competition is particularly interesting. The main result is that, when the total number of firms in the industry is kept fixed and one group of firms becomes larger (stronger intra-brand competition), there is a positive impact on the individual output of a member firm when the size of its group and/or the degree of horizontal product differentiation are large enough. In such a case, the positive effect of inter-brand competition overcomes the negative effect of a stronger intra-brand competition.

[2] This model launched an important stream of the literature that compares the competitiveness of Cournot and Bertrand solutions when products are differentiated. This paper has been extended in many ways (see, e.g. Vives 1985, Okuguchi 1987, Häckner 2000, and Amir & Jin 2001) with the objective of analyzing the same core fundamental question, that is, which solution concept provides a more competitive outcome.

[3] Note that our model is different from the one in Tanaka (2001), where two groups of firms, characterized by different production costs, produce N varieties, and where the model evolves under an imitative rule with mutations.

We then turn to the dynamic mixed differentiated oligopoly and look for its steady-state composition. The long-run composition of a differentiated oligopolistic industry under a replicator dynamics assumption has been investigated in the literature, albeit in different settings. Examples include Bischi, Lamantia & Radi (2013), where a differentiated oligopoly is used to describe a fishing industry where fishers can periodically choose to harvest either one of two fish species differing in harvesting cost and selling price; Kopel, Lamantia & Szidarovszky (2014), which proposes a differentiated oligopoly model where profit-maximizing firms compete in quantities against socially concerned firms, and where firms can periodically decide to change their utility function; Hu et al. (2014), which uses a horizontally differentiated oligopoly to study competition when "green" firms penetrate a mature market populated by "brown" firms. Our main results can be summarized as follows: in a differentiated oligopoly with two varieties, a single steady state exists, and the long-term composition of the industry (only brown, only green, or mixed firms) can be characterized according to the value of the model parameters.

In the second part of the paper, we apply our model to the specific case of the sustainable wine industry.[4] The coexistence of sustainable and conventional wine production is represented by the differentiated oligopoly, and the dynamic composition of the oligopoly captures both the possibility for a wine producer to periodically revise her production practice decision and the periodic auditing in which sustainable wine producers must verify their compliance with program standards.[5]

The other features of the sustainable wine industry translate into specific assumptions about the parameter values of the general model. As mentioned earlier, an important component of the sustainable wine industry is the presence of certification costs, which are a significant factor when deciding whether to participate in the sustainable wine industry.[6] In our model, firms that want to obtain a label validating the sustainability of their product must pay an additional fixed cost over any fixed cost incurred by conventional wine producers.

In general, the production methods implemented in the sustainable industry result in a

[4]The "greenification" of the wine industry has mainly been studied from an empirical perspective. In particular, this literature examines how firms understand the concept of sustainability (Szolnoki 2013), what drives and prevents them from adopting sustainable practices (Marshall, Cordano & Silverman 2005; Gabzdylova, Raffensperger & Castka 2009; Berghoef & Dodds 2013) and what are the specific features of firms that are keen to implement this new business practice (Barisan et al. 2016).

[5]The CSWA accepts applications for certification on a rolling basis, with three certification deadlines per year, while SIP allows yearly applications.

In terms of auditing their members to allow them to keep the certifying label, both the SIP and CSWA programs assess their winery and vineyard members annually; however, the SWNZ and Sustainable Wine South Africa (SWSA) audit their members every three years, and the National Sustainability Code for Chilean wines reviews its members every two years.

[6]See for instance CCSW (2018), p. 28: "We are very sensitive to the fact that the cost of certification is an important deciding factor for potential participants..."

higher marginal production cost than in the conventional wine industry. However, for the members of a certification program, there exists the possibility of sharing knowledge about sustainable production practices, with the aim of improving the quality of sustainable wine and of reducing its production cost (see, for example, the workshop calendar on the CSWA website). This aspect is modelled in a second stage as an alternative cost structure for green producers, wherein their production cost is decreasing in the number of participants in the sustainable program.

Finally, on the consumer side of the market, we assume that consumers have a taste for variety and buy both sustainable and conventional wines. Moreover, as shown in many studies,[7] we assume that consumers are willing to pay a premium price for wine that has been certified sustainable. We investigate two different assumptions about the structure of this premium price, that is, a constant premium or one that depends on the recognition attached to the ecolabel, as measured by the number of participating firms.

The main findings of this second part are that the results of the general model can be directly applied to a sustainable wine industry when the model parameters are constant, and that the general model is able to replicate some real-life examples. Furthermore, when some parameter values depend on the number of green firms, as, for instance, when the green premium price depends on the ecolabel notoriety or when the production costs decrease with the number of firms participating in a certification program, we find that multiple steady states may occur and that initial conditions become crucial in determining the long-run composition of the industry.

We conclude the paper with a welfare analysis of the different possible steady-state industry compositions and where they stand as compared to the welfare-maximizing industry composition. After identifying the conditions under which a green industry is socially preferred over a brown industry, we observe that the long-run composition of the industry obtained when the model parameters are not state-dependent is very close to the welfare-maximizing industry composition; however, when the model parameters are state-dependent, the steady-state industry composition can be very far from the welfare-maximizing one, opening up space for political intervention.

The paper is organized as follows. Section 2 presents the generic differentiated oligopoly model with K varieties produced by N firms. Section 3 analyzes the special case of an oligopoly producing two varieties ($K = 2$), discussing the impact of substitutability and of competitive pressures, and the steady-state composition of such an industry. Section 4 specializes the results to the case of the wine industry where vineyards and wineries produce

[7] Evidence from Spain is provided in Sellers (2016) and in Sellers-Rubio & Nicolau-Gonzalbez (2016); from New Zealand in Forbes et al. (2009); from Italy in Vecchio (2013), in Pomarici & Vecchio (2014), and in Sogari, Mora & Menozzi (2016a) and (2016b). See also Schäufele & Hamm (2017) for a review.

conventional or sustainable wine, and where some parameter values may depend on the number of sustainable-certified producers. Section 5 discusses welfare considerations, and Section 6 is a conclusion.

2 A general differentiated oligopoly model

Consider an industry populated by N firms. Producers are divided into K groups of similar types, and members of the same group use the same technology to produce a homogeneous product. Denote by G_k the set of producers of type $k = 1, ...K$ and by $n_k = |G_k|$ the number of producers within group k. Accordingly, the total production cost of a quantity q_{ki} of product k by producer $i \in G_k$, $k = 1, ...K$ is given by

$$C_{ki} = f_k + m_k q_{ki}.$$

Since goods produced by firms of a given type are homogeneous, consumers are offered K product varieties. We assume that the representative consumer has a taste for variety, and her quadratic utility function is described by

$$U(\mathbf{q}) = \sum_{k=1}^{K} A_k Q_k - \frac{1}{2} \sum_{k=1}^{K} \sum_{j=1}^{K} D_{kj} Q_k Q_j,$$

where the A_k are positive constants and $Q_k \equiv \sum_{i \in G_k} q_{ki}$ is the total production of the firms of type k. The matrix $[D_{kj}]$ is symmetric, with $D_{kk} > 0$, $D_{kj} \geq 0$,[8] and is assumed positive definite, while the vector $A = [A_k]$ is assumed to satisfy $D^{-1}A \geq 0$. Under these assumptions, the representative consumer's utility function is concave and its maximum value is attained for non-negative quantities. In the same way as in Häckner (2000), the parameters A_k can be interpreted as the quality (vertical) differentiation between the product varieties. For $j \neq k$, D_{kj} is the symmetric degree of substitutability between any pair of varieties. When $D_{kj} = 0$, products k and j are completely independent; if $[D_{kj}]$ is diagonal, each group of producers of a given type becomes an independent oligopoly selling a homogeneous product.

The representative consumer's utility-maximisation problem is then

$$\max \{U(\mathbf{q})\} \text{ s.t. } \sum_{k=1}^{K} P_k Q_k \leq I,$$

where P_k is the price of variety k, and I is the total budget. Consequently, the inverse

[8] Goods can be either substitutes or independent.

demand function faced by producers of type k is given by

$$P_k = A_k - \sum_{j=1}^{K} D_{kj} Q_j. \tag{1}$$

For $i \in G_k$, we denote by E_k the quantity $A_k - m_k$, which is assumed to be strictly positive. The parameter E_k depends on quality and cost parameters and can be interpreted as an indicator of efficiency. For example, if $E_k > E_j$, k-type firms are more efficient than j-type firms; this greater efficiency can result from a better quality and/or from a cost advantage. Producers compete in quantities, both within each group, by selling a homogeneous product, and with producers of other groups, by offering a different variety. The optimization problem of a single representative producer $i \in G_k$ is then given by

$$\begin{aligned} & \max_{q_{ki} \geq 0} \{\pi_{ki} = P_k q_{ki} - C_{ki}\} \\ = & \max_{q_{ki} \geq 0} \left\{ \left(E_k - \sum_{j=1}^{K} D_{kj} Q_j \right) q_{ki} - f_k \right\}. \end{aligned}$$

This is a convex optimization problem and, assuming an interior solution, the optimal quantity for Producer i is obtained from the first-order condition:

$$E_k - D_{kk} q_{ki} - \sum_{j=1}^{K} D_{kj} Q_j = 0. \tag{2}$$

Since producers in the same group have identical parameters, the equilibrium solution of the oligopoly game is obtained by simultaneously solving the following linear system of equations, assuming equilibrium quantities are positive:

$$E_k - D_{kk} q_k - \sum_{j=1}^{K} D_{kj} n_j q_j = 0, \;\; k = 1, ..., K. \tag{3}$$

Using (1), the unit margin for a producer of type k is given by

$$\begin{aligned} P_k - m_k &= E_k - \sum_{j=1}^{K} D_{kj} n_j q_j \\ &= D_{kk} q_k \end{aligned}$$

and the equilibrium profit for a producer of type k is then

$$\begin{aligned} \pi_k^* &= (P_k - m_k)\, q_k - f_k \\ &= D_{kk} q_k^2 - f_k. \end{aligned} \tag{4}$$

We retrieve the differentiated oligopoly model of Amir & Jin (2001) when $n_k = 1$ and $f_k = 0$ for $k = 1, ..., K$ and the differentiated oligopoly of model of Häckner (2000) by setting, for $k = 1, ..., K$, $n_k = 1$, $D_{kk} = 1$, $D_{kj} = D$ for $j \neq k$ and $f_k = 0$. By setting $K = 2$, $n_k = 1$ and $f_k = 0$ for $k = 1, 2$, we obtain the differentiated duopoly model of Singh and Vives (1984). In all three cases, the oligopoly model is used to compare Bertrand and Cournot competition.

The market model proposed in Kopel et al. (2014), with two different groups of firms producing two different types of products ($K = 2$), can be embedded in our general model with $f_k = 0$; however the equilibrium solution of the oligopoly game obtained by the authors differs from (3) because the model assumes that producers in the "socially concerned" group pursue an objective that is not pure profit but that also includes a consumer surplus term.

The market model described in Hu et al. (2014) also has two groups of firms producing either ordinary or green products, but it does not assume symmetrical substitutability parameters.[9] It can be nested in our general model if $D_{kj} = D_{jk}$ and $f_k = 0$.

In the following section, we solve this general oligopoly model for $K = 2$ and show that the equilibrium solution of this general differentiated oligopoly model, where competition is present within groups of firms selling a homogeneous product and across different substitutable varieties, presents interesting characteristics that differ from the solution of the classical differentiated oligopoly model with $n_k = 1$ for $k = 1, ..., K$.

3 A general oligopoly with two varieties

We now compute the equilibrium solution of the oligopoly game for the specific case where players are divided into two groups ($K = 2$), labeled "green" and "brown," and $k \in \{G, B\}$.[10] Let $F_G \equiv D_{GG} > 0$, $F_B \equiv D_{BB} > 0$ and $S \equiv D_{GB} = D_{BG}$. The assumptions of strict concavity of the representative consumer's utility function implies that $S^2 < F_G F_B$ and the assumption that the maximum utility is achieved in the positive quadrant corresponds to $S \leq \min\left\{F_G \frac{A_B}{A_G}, F_B \frac{A_G}{A_B}\right\}$.

[9] Note that symmetry of the substitutability parameters obtains when demand functions are derived from a representative consumer's utility function.

[10] Note that, in this section, no assumption is made about the relative size of the model parameters in the green and brown industries, so that the green and brown labels are commutable.

The solution of the system (3) yields

$$q_G = \frac{F_B E_G (n_B + 1) - S E_B n_B}{\Omega} \tag{5}$$

$$q_B = \frac{F_G E_B (n_G + 1) - S E_G n_G}{\Omega} \tag{6}$$

$$\Omega = F_G F_B (n_G + 1)(n_B + 1) - S^2 n_G n_B > 0. \tag{7}$$

Further assume that $S < \min \left\{ E_B \frac{F_G}{E_G}, E_G \frac{F_B}{E_B} \right\}$ and that $f_k < F_k q_k^2$ for $k \in \{G, B\}$. This is sufficient to ensure that both types of players participate in the market at equilibrium. This result generalizes the Cournot solution found in Singh and Vives (1984) for the duopoly case ($n_B = n_G = 1$).

We first investigate how the individual equilibrium quantities respond to changes in the parameters related to both the market demand and composition. We then characterize the steady-state composition of the industry when producers can move from one variety to the other.

3.1 Impact of demand function parameters

It can easily be shown that there is a negative relationship between the equilibrium output of a firm and the slope of its inverse demand function, while there is a positive relationship between the equilibrium output of a firm and the slope of the inverse demand function for the other variety:

$$\frac{\partial q_k}{\partial F_k} = -F_j q_k (n_k + 1) \frac{n_j + 1}{\Omega} < 0$$

$$\frac{\partial q_k}{\partial F_j} = S n_j \frac{n_j + 1}{\Omega} q_j > 0, \ k, j \in \{B, G\}, \ k \neq j.$$

This result can be explained by examining what happens to the consumer demand: for a given set of prices, an increase in the sensitivity of consumers to the price of product k contracts their demand for that product

$$Q_k = \frac{F_j (A_k - P_k) - S (A_j - P_j)}{F_k F_j - S^2}$$

$$\frac{\partial Q_k}{\partial F_k} = -\frac{Q_k F_j}{F_k F_j - S^2} < 0, \ k, j \in \{G, B\}, \ j \neq k,$$

while an increase in consumer sensitivity to the price of the alternative variety j expands their demand for product k

$$\frac{\partial Q_k}{\partial F_j} = \frac{SQ_j}{F_j F_k - S^2} > 0,\ \ k, j \in \{G, B\}\,,\ \ j \neq k.$$

The relationship between the *degree of substitutability* S and the individual and total equilibrium quantities is more complex and depends both on the market parameters and the industry composition (see Appendix 7.1). An increase in S can be interpreted as a more intense horizontal product competition.

When S is small, that is, when the two markets are relatively independent, an increase in the degree of substitutability decreases the equilibrium quantities of both varieties, as expected. However, for a sufficiently large S (more interconnected markets), under mild conditions on the parameter values (see 17), a further increase in the degree of substitutability has a positive impact on the equilibrium quantity of the variety k satisfying

$$\frac{F_k}{E_k^2} \frac{n_k + 1}{n_k} < \frac{F_j}{E_j^2} \frac{n_j + 1}{n_j},\ \ j, k \in \{G, B\}\,,\ \ j \neq k \tag{8}$$

and a negative impact on the equilibrium quantity of the other variety.

Finally, we find that the total output is increasing in S for large enough S if both conditions (8) and

$$F_k \frac{n_k + 1}{n_k} > F_j \frac{n_j + 1}{n_j},\ \ j, k \in \{G, B\}\,,\ j \neq k$$

are satisfied for the variety with the highest efficiency ($E_k > E_j$).

The impact of changes in the degree of substitutability between the two product varieties has also been investigated in other works but with simplifying assumptions. Our result generalizes that of Dou & Ye (2017), which considers the special case where $F_G = F_B$ and $E_G = E_B$, and shows that the output of a firm belonging to the largest group increases with S when S is large enough.

It is interesting to contrast this finding with the corresponding result obtained in Kopel et al. (2017) in a duopoly model ($n_G = n_B = 1$) under the simplifying assumption $F_G = F_B = 1$ and $E_G = E_B = E$. In Kopel et al. (2017), the equilibrium output is decreasing for both varieties. However, when we move to a context with inter- and intra-group competition, we find that, if $n_k > n_j > 1$, the equilibrium output of Firm k is increasing in S for

$$S \in \left(\frac{n_j + 1}{n_j} - \frac{\sqrt{n_k\,(n_j + 1)\,(n_k - n_j)}}{n_k n_j}, 1 \right),\ \ j, k \in \{G, B\}\,,\ j \neq k.$$

On the other hand, when $F_G = F_B = F$, the total equilibrium output is always decreasing in S, and the result obtained in Theilen (2012) for a duopoly model still holds.

Note that, for the specific application to the wine industry, the quality and production cost parameters (E_G and E_B), as well as the sensitivity of consumers to sustainable and conventional wine (F_G and F_B) are expected to differ, as will be discussed in Section 4.

3.2 Impact of industry composition and size

We now assess how the individual equilibrium quantity responds to changes in the industry composition and/or size. In particular, we focus on the degree of intra-brand competition under two different scenarios. In the first case, we consider an increase in the number of firms in one group, which leaves the number of firms in the other group unchanged, so that the total number of firms increases. This scenario can be assimilated to long-term structural changes in industry size and composition. Under the second scenario, we assume that the total number of firms in the industry is fixed, so that an increase in the number of firms in one group is compensated by a decrease in the number of firms in the other group. This scenario can be assimilated to short-term changes in industry composition. Finally, we consider the possibility of an increase in the total number of firms, where the proportion in each group remains constant.

In the first scenario, it is straightforward to check that a unilateral increase in the number of firms in a given group has a negative impact on the individual output of all firms in the industry. This is due to a general intensification of competition. However, the impact of a unilateral increase in the number of firms in a given group on the total output of each group is different: the total equilibrium quantity of the group that experiences growth increases (due to the greater size), but the total equilibrium quantity of the competing group decreases. When we look at the total industry quantity Q, the impact of increasing the size of group k when the size of group j does not change depends on the degree of substitutability S. If the degree of substitutability is relatively low ($S < \frac{F_j(n_j+1)}{n_j}$), that is, if the markets for the two varieties are relatively independent, the total industry quantity increases. The reverse is true when the degree of substitutability is relatively high ($S > \frac{F_j(n_j+1)}{n_j}$), that is, when the markets for the two varieties are more interconnected. Note that the threshold value of S, at which the impact on the total output changes, is decreasing in n_j (see Appendix 7.2.1).

If, however, N is assumed constant, then the impact of an increase in the number of firms within group $k \in \{G, B\}$ is given by

$$\frac{dq_k}{dn_k} = \frac{\partial q_k}{\partial n_k} - \frac{\partial q_k}{\partial n_j} \text{ with } j \in \{G, B\} \text{ and } j \neq k,$$

where $\frac{\partial q_k}{\partial n_k}$ represents the marginal impact of intra-group competition and $\frac{\partial q_k}{\partial n_j}$ represents the marginal impact of inter-group competition. Note that, when N is constant, the impact of an increase in n_k is equal to the impact of a decrease in n_j.

As a result, we find that an increase in the number of firms in a given group can have a positive impact on the individual output of firms in that group if S and/or the size of the group are large enough (see Appendix 7.2.2). The interpretation is that, when the degree of horizontal product differentiation is small, so that products are perceived as similar and markets are tightly linked, a large group can positively impact the inter-group competition to overcome the negative impact of intra-group competition.

If we adopt the simplifying assumptions of the model of Dou & Ye (2017), i.e., $F_G = F_B = F$, $E_G = E_B = E$, we find that an increase in the number of firms in group k has a positive impact on q_k when $n_k > \frac{1}{4}(3N+1)$ and S is large enough.

Finally, it is straightforward to show (see Appendix 7.2.3) that a proportional increase in the number of firms in both groups decreases the output of all firms in the market.

3.3 Steady state market composition

Note that when the fixed cost of production does not differ across producer types, the highest profit in Cournot competition is achieved by the players with the highest $F_k q_k^2$. This is no longer the case when the production of different varieties generates different fixed production costs. Then, comparing the profits of groups of players becomes a more complex problem. For the specific application we are considering here, opting for a sustainable production technology involves an additional certification cost that is independent of the quantity produced.

The equilibrium quantity, and therefore the profit of both kinds of producers, depends on the composition of the industry, which, for a fixed N, can be characterized by the number of green producers, denoted by $n \equiv n_G$. We now consider the possibility that producers may decide to change their production technology, so that n evolves over time according to evolutionary pressures in favor of the relatively better-performing group. For instance, we can assume that the number of green firms changes following the standard replicator dynamics

$$n(t+1) = N\frac{n(t)\pi_G^*(n(t))}{n(t)\pi_G^*(n(t)) + (N-n(t))\pi_B^*(n(t))},$$

or any evolution process such that the number of green producers increases (*resp. decreases*) when the profit of green producers is higher (*resp. lower*) than that of brown producers. Assuming that N is sufficiently large, we define the continuous extensions $\pi_k : [0, N] \to \mathbb{R}$, $k \in \{G, B\}$ of the equilibrium profit of both kinds of producers as a function of n. A steady state of this dynamic process is a number n^* such that

$$\pi_G(n^*) = \pi_B(n^*), \tag{9}$$

that is,

$$\begin{aligned} F_G\left(F_B E_G\left(N-n^*+1\right)-S E_B\left(N-n^*\right)\right)^2-F_B\left(F_G E_B\left(n^*+1\right)-S E_G n^*\right)^2 & \\ -\delta\left(F_G F_B\left(n^*+1\right)\left(N-n^*+1\right)-S^2 n^*\left(N-n^*\right)\right)^2 &= 0 \end{aligned} \tag{10}$$

where $\delta = f_G - f_B$. Without loss of generality, we assume that $\delta \geq 0$, that is, green firms are identified with the firms that have the highest fixed cost.

We analytically derive conditions, on the fixed cost difference δ, under which different compositions of the industry arise at the steady state, as stated in the following proposition.

Proposition 1 *Define*

$$\begin{aligned} \lambda_1 &\equiv \frac{F_B F_G E_G^2 - \left(NSE_G - F_G E_B\left(N+1\right)\right)^2}{F_B F_G^2\left(N+1\right)^2} \\ \lambda_2 &\equiv \frac{\left(NSE_B - F_B E_G\left(N+1\right)\right)^2 - F_B F_G E_B^2}{F_B^2 F_G\left(N+1\right)^2}. \end{aligned}$$

If $0 \leq \delta \leq \lambda_1 < \lambda_2$, then, at the steady state, the industry is populated by green firms only.

If $\lambda_1 < \delta < \lambda_2$, then there exists a single steady state where green and brown firms coexist.

If $\lambda_2 \leq \delta$, then, at the steady state, the industry is populated by brown firms only.

Proof. See Appendix 7.3. ■

Note that the equilibrium solution when the industry is populated by a single type of firm (brown or green) is readily obtained by setting $n_G = N$ or $n_B = N$ in (5)-(7). This solution yields a positive quantity and a positive profit for the firms that are not present in the market, which are interpreted as the limit quantity and profit when the number of firms of one type vanishes.

It is easy to check that λ_1 and λ_2 are both increasing in F_B and E_G and decreasing in F_G and E_B, while λ_1 (*resp.* λ_2) is increasing (*resp. decreasing*) in S (see Appendix 7.4.1). From the results of Proposition 1, comparative statics allows us to derive the following properties.

P1 Starting from an industry with only brown firms, green firms appear with increases in λ_2 and decreases in δ, that is,

1. decreases in S (more horizontally differentiated products);
2. increases in F_B (*resp. decreases in* F_G) (lower (*resp. higher*) impact of price on demand for brown (*resp. green*) products);
3. decreases in m_G (resp. increases in m_B) (more (*resp. less*) cost-efficient green (*resp. brown*) production technology);
4. increases in $A_G - A_B$ (higher choke price for green products; more vertically differentiated products);
5. decreases in δ (smaller (*resp. greater*) fixed cost for green (*resp. brown*) producers).

Moreover, we derive the following property by studying the effects of parameter changes on the functions on the stability condition (10) (see Appendix 7.4.2).

P2 Starting from a mixed industry, more green firms appear with

1. increases in E_G (*resp. decreases in* E_B) (changes in quality and/or cost efficiency), or
2. decreases in δ,

while changes in other parameter values impact the number of green firms in an ambiguous way.

4 The sustainable wine industry

We now investigate whether the results found for a general oligopoly with two varieties can be further refined in the specific case of the sustainable wine industry. Considering the wine industry translates into making specific assumptions about the parameter values.

In this section, green firms represent the sustainable wine producers, and brown firms represent the conventional wine producers (vineyards and wineries). The adoption of sustainable practices is captured by the marginal cost m_G and the fixed cost f_G.

The marginal cost m_G applies to sustainable production methods, such as choosing hand-picking over machine harvesting, installing owl boxes rather than using poison for rodent control, using biodiesel in the tractors, powering the winery with solar energy, or using lighter-weight glass. We assume that

$$m_G > m_B,$$

that is, that the technology implemented to produce sustainable wine is more expensive than the one adopted to make conventional wine.[11]

Moreover, vine growers and wine makers who want an official seal of sustainability to affix to their bottles must go through a certification process offered by a certification program. This certification process generates a fixed cost that includes the auditor fees and any administration/membership fee to the organization managing the certification program. A fixed fee is also paid by firms that have already been approved for a sustainability label but need to be periodically audited to maintain their status.[12] These fixed costs add up to any other fixed costs borne by a conventional wine producer, so we assume that

$$f_G > f_B.$$

On the demand side, we make no prior assumptions on the relative sensitivity of consumers to the price of each wine variety, that is,

$$F_G <=> F_B.$$

As reported in the literature (see Schäufele & Hamm 2017 for a review), we assume that consumers are willing to pay a premium price for a wine that has been certified sustainable. For instance, in the exploratory study of consumer demand for sustainable wine in New Zealand, Forbes et al. (2009) report that 73% of respondents were willing to pay a premium price for an environmentally sustainable wine and that this premium would be up to 5% for one-third of the participants, and between 6% and 11% for another third. We model this green premium by assuming that when a quantity $Q + Q'$ is produced, where Q is the total quantity of variety k and Q' is the total quantity of the other variety, the price of the green variety is higher, for any feasible Q and Q' :

$$A_G - F_G Q - SQ' > A_B - F_B Q - SQ',$$

which translates into a higher choke price, that is,

$$A_G > A_B,$$

[11] See Castellini et al. (2017) for an estimate of the production costs of biodynamic grapes and wine.

[12] In the CCSW program, fixed costs include, for instance, an annual administration fee paid to the certifying program and auditor fees paid to a third-party auditor; in the SIP program, applicants pay a one-time application fee, inspector fees and a licensing fee per acre or gallon.

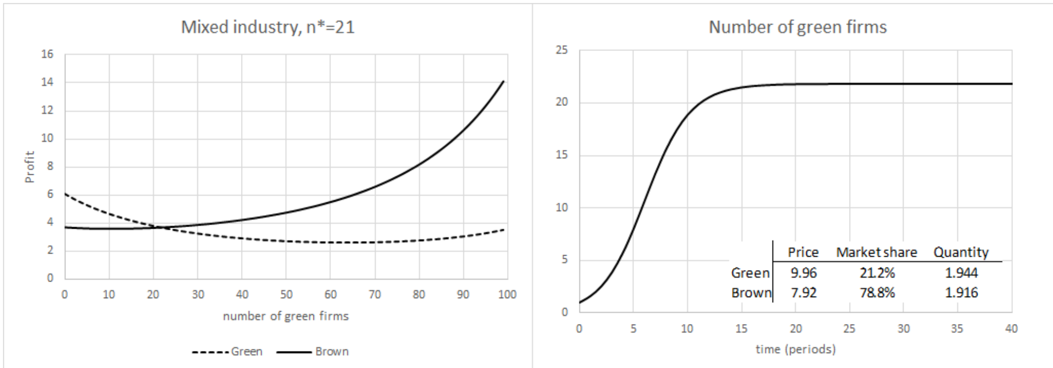

Figure 2: Steady state with a mixed industry. Parameter values are $N = 100$, $S = 0.9985$, $F_G = 1.01F_B$, $A_G = A_B + 2.29$, $m_G = 8$, $m_B = 6$, $f_G = 0.08$.

with the additional condition

$$A_G F_B > A_B F_G,$$

which is always satisfied if $F_B \geq F_G$.

4.1 Constant parameter values

In this section, we assume that all model parameter values are independent of the market composition n. In particular, the difference between the choke prices for sustainable and conventional wines is a constant positive *premium price*, denoted p, such that

$$A_G = A_B + p.$$

Given these assumptions on the parameters, the stability condition (21) and the comparative statics results remain valid, and we can find numerical examples satisfying all three possibilities listed in Proposition 1 for the steady-state industry composition.

We now provide some numerical illustrations that represent different industry compositions. For comparison purposes and without loss of generality, we normalize the values of parameters A_B, F_B and f_B in all numerical experiments, so that $A_B = 200$, $F_B = 1$ and $f_B = 0$.

Figure 2 is an example of an industry where sustainable wine practices have spread moderately, as in, e.g., California, where 21% of the total farmed acreage was CCSW certified in 2017. In this example, on the demand side of the market, consumers are less sensitive to the price of sustainable wine than to that of regular wine ($F_G = 1.01F_B$), products are highly substitutable (large S), and the green premium amounts to 1.1% of the choke price. On the supply side, the marginal production cost of green wine is 33.3% higher, while the

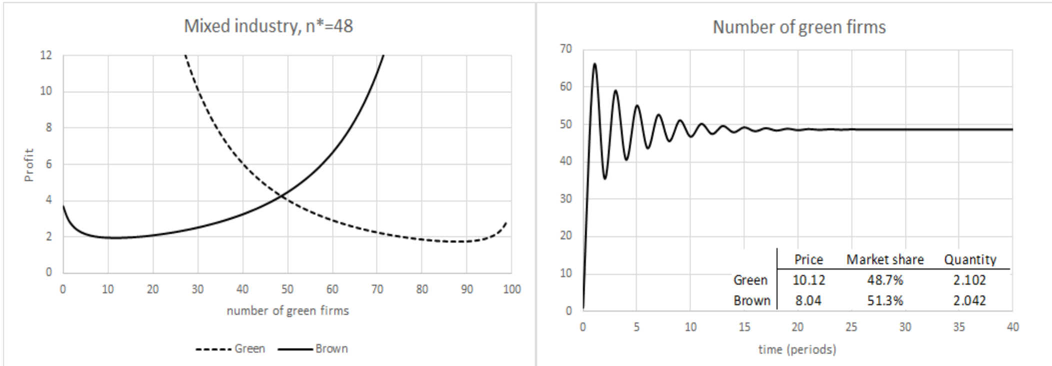

Figure 3: Steady state with a mixed industry. Parameter values are $N = 100$, $S = 0.85$, $F_G = 1.01F_B$, $A_G = A_B + 2.29$, $m_G = 8$, $m_B = 6$, $f_G = 0.08$.

additional fixed costs borne by sustainable wine producers ($f_G = 0.08$) amount to 2.1% of the equilibrium profit. At equilibrium, 21 green firms produce 21% of the total quantity, the price of sustainable wine is 26% higher than that of regular wine, and each green firm produces 1.5% more in terms of quantity than a brown firm.

Figure 3 illustrates the impact of greater horizontal product differentiation (smaller S). This could be the result of some (exogenous) investment to make consumers more aware of the difference between the two products, like a more distinctive label or an advertising campaign focused on sustainable winegrowing practices.[13] In this example, all parameter values are the same as in Figure 2, except that $S = 0.85$ (note that this case falls within Statement 3 of Property P2 in Section 3.3). This results in a higher number of green firms as well as a higher profit, selling price, and production quantity for both kinds of producers. At equilibrium, 48 green firms produce 48.7% of the total quantity, the price of sustainable wine is 26% higher than that of conventional wine, and each green firm produces 2.9% more in terms of quantity than a brown firm. Moreover, at equilibrium, the slopes of both profit functions are steeper than in the preceding example, which results in an oscillating trajectory to the steady state under the replicator dynamics.

The transition from the example depicted in Figure 2 to the one depicted in Figure 3 can be explained in light of the results obtained for the general model in Section 3.1. When we apply the conditions stated in Appendix 7.1 to this specific case, we obtain that, at $n_G = 21$, $\frac{F_B E_G^2}{F_G E_B^2} < \frac{n_B(n_G+1)}{n_G(n_B+1)}$ so that $\Delta_G < 0$ and $\Delta_B > 0$ and we can conclude that a decrease in S will increase the equilibrium quantity q_G. To understand what happens to q_B, we verify that, at

[13] See, for instance, the Sonoma County Winegrowers' national advertising campaign on vine balance, using content taken directly from the *California Code of Sustainable Winegrowing Workbook*, which appeared in the November 2014 issues of *Food & Wine*, *Wine Spectator*, and *Wine Enthusiast Magazine*.

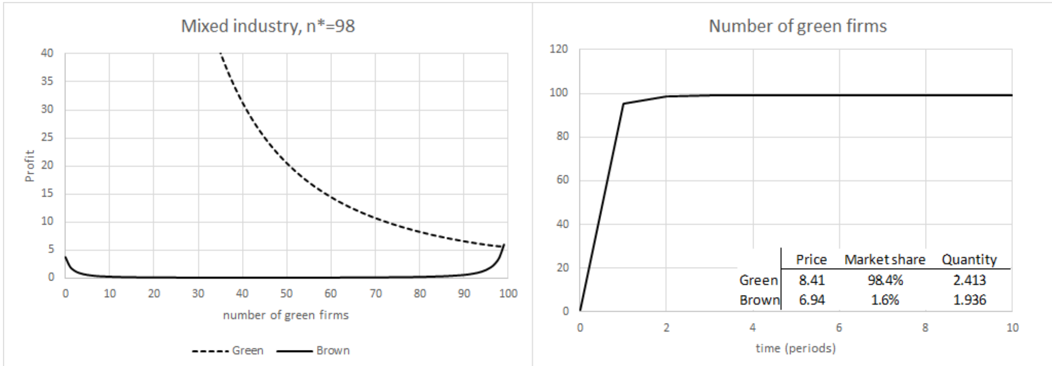

Figure 4: Steady state with a mixed industry. Parameter values are $N = 100$, $S = 0.8$, $F_G = F_B$, $A_G = A_B + 48$, $m_G = 6$, $m_B = 5$, $f_G = 0.08$.

$n_G = 21$,

$$F_G \frac{E_B}{E_G} \frac{n_G + 1}{n_G} - \frac{\sqrt{\Delta_B}}{2E_G n_G n_B} < S < F_G \frac{E_B}{E_G} \frac{n_G + 1}{n_G},$$

which implies that a marginal decrease in S reduces the equilibrium quantity of brown firms. As a result, green firms experience an increase in their profits, while the profit of brown firms deteriorates. According to the replicator dynamics, some brown firms will then switch to sustainable wine practices, until a new steady-state industry composition is attained, with 48% of green wine producers.

Figure 4 exemplifies a wine industry with a very high penetration of sustainable production practices, akin to the situation in New Zealand, where, in 2016, 98% of New Zealand's vineyard producing area was SWNZ certified. The broad participation of sustainable wine producers is obtained for a constellation of parameters that depicts a market where products are highly substitutable (large S), the green premium amounts to 24% of the choke price, and the impact of price on demand is the same for both varieties. The marginal production cost of sustainable wine is not as high as in the previous examples (20% higher than the marginal production cost of conventional wine), while the additional fixed cost is the same. At equilibrium, 98 green firms produce 98.4% of the total quantity, the price of sustainable wine is 21% higher than that of regular wine, and each green firm produces 25% more wine than a brown firm.

A wide diffusion of sustainable production practices can also be observed under a different market demand configuration. In Figure 5, the number of green wine producers at the steady state is still 98 out of 100. While the cost parameters are the same as in the previous example, this wine industry shows a lower substitutability between sustainable and conventional wines (smaller S); brown consumers who are less sensitive to the price of conventional wine than green consumers to the price of sustainable wine ($F_B = 1.5F_G$); and a very large green

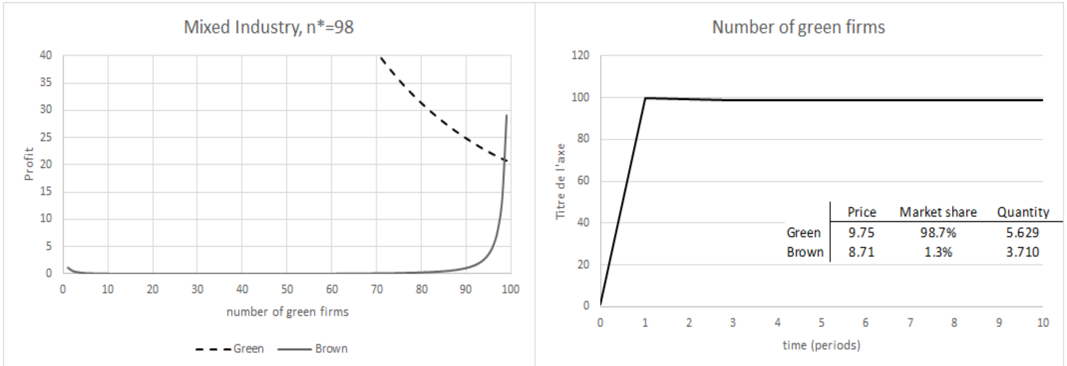

Figure 5: Steady state with a mixed industry. Parameter values are $N = 100$, $S = 0.3333$, $F_G = F_B/1.5$, $A_G = A_B + 180$, $m_G = 6$, $m_B = 5$, $f_G = 0.08$.

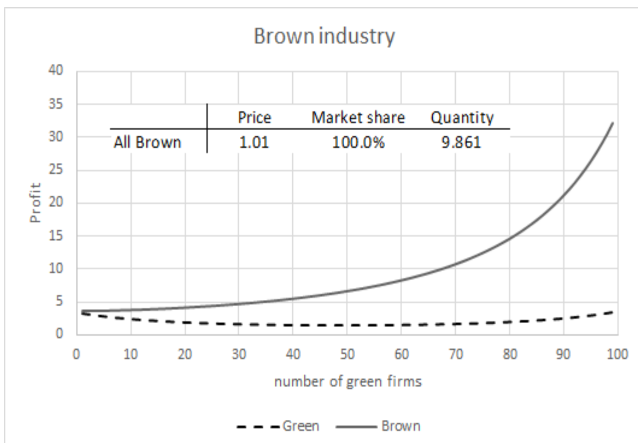

Figure 6: Brown industry. Parameter values are $N = 100$, $S = 0.99$, $F_G = F_B$, $A_G = A_B + 1$, $m_G = 8$, $m_B = 5$, $f_G = 0.08$.

premium (90% of the choke price). At the steady state, the 98 green firms produce 98.7% of the total quantity, the price of sustainable wine is 10.7% higher than that of regular wine, and each green firm produces 52% more wine than a brown firm.

Finally, Figure 6 illustrates the case where no green firm enters the market. In this particular example, products are highly substitutable, the marginal cost of production is 60% higher in the green industry, and the premium price is only 0.5% of the choke price.

Our numerical experiments show that, given the specific features of the sustainable wine industry, the most important parameters affecting the participation of sustainable wine producers are the parameters of the demand function. When F_G is close to F_B, the most important factor is the premium price. Substitutability between the two products can also play an important role, while the composition of the industry is less sensitive to cost parameters. Changes in the fixed certification costs result in a vertical shift of the green firms' profit function. As can be observed in Figures 2 to 6, within a reasonable range, such a shift does not significantly impact on the penetration of green firms. On the other hand, the impact of an increase in the production cost of green firms depends on the degree of substitutability between products; it can be important when products are highly substitutable, but is less significant for horizontally differentiated products.

4.2 Impact of market composition

We now consider the possibility that some market parameter values depend on the market composition, that is, on the relative number of green producers. Such a dependency could happen, for instance, when the choke price, or the size of the market for the green product, increases with the number of green producers. As suggested in Lozano, Blanco & Rey-Maquieira (2010), well-known labels generate greater consumer trust in the product, so that consumers are willing to pay a higher premium price. This "reputation effect" can be captured by linking the premium price to the number of sustainable wine producers (a broader presence of certified sustainable wines helps consumers learn about the label).

On the other hand, one of the advantages put forward by certification agencies is the possibility for members to share knowledge about sustainable production methods, with the aim of improving sustainable wine quality and reducing its production cost. Green production costs could therefore decrease with the number of green producers, due to knowledge spillovers.

More specifically, we consider the case where E_G (that is, quality and/or cost efficiency) increases linearly with n, and we study the effect of this assumption on the steady-state composition of the industry.

Define

$$E_G(n) = E_0 + n\frac{E_N - E_0}{N} \tag{11}$$

where $E_N > E_0 > 0$ and E_N (*resp.* E_0) represents the limit of the parameter $E_G(n)$ when the number of brown (*resp.green*) producers vanishes.

Under Assumption (11), it is no longer possible to characterize the steady-state industry composition analytically. Numerical experiments show that all three single-steady-state cases identified in Section 3.3 can happen (all-green, all-brown, or mixed). Moreover, when E_G depends on n, multiple steady states can coexist, so that different steady-state scenarios can emerge according to the initial conditions. More particularly, it is possible to obtain examples where, depending on the initial conditions, the industry can turn into either a brown industry or a mixed industry, or into either a brown or a green industry (polarized market).

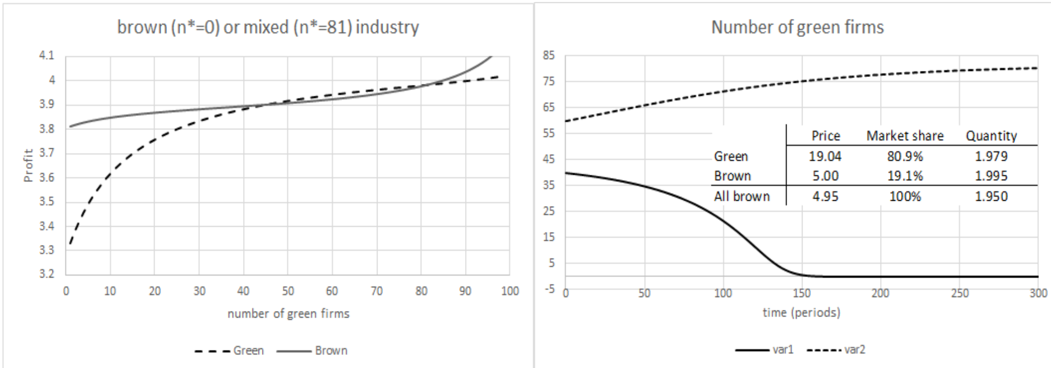

Figure 7: Efficiency E_G increasing with the number of green producers, with two coexisting steady states, $n^* = 81$ and $n^* = 0$, according to the initial conditions. Parameter values are $N = 100$, $S = 0.98$, $F_G = 1.03\ F_B$, $E_B = 197$, $E_N = 206.95$, $E_0 = 193$, $f_G = 0.05$. This could correspond, for instance, to an increase in A_G from 210 to 210.1 and a decrease in m_G from 17 to 3.15 as n varies from 0 to 100.

Figure 7 illustrates a case where the two coexisting steady states consist of only brown firms or of mixed types. The basin boundary of the initial states generating trajectories converging to a mixed industry is given by $n = 45$, meaning that if at least 45 wine producers adhere to the sustainability concept, then others will join the program over time due to their relatively better performance compared to conventional wine producers, so that, in the long run, there will be 81 sustainable wine producers. In that example, decreasing F_G so that consumers' price sensitivity for the two kinds of wine becomes more similar or reducing the difference in marginal production costs reduces the initial number of firms required to attain a mixed industry and increases the number of green firms in the mixed steady state, until the steady state is a market containing only green firms.

Figure 8 provides an example where the wine market can only be populated by either conventional or sustainable wine producers. The threshold value that makes the difference in the long-run composition of the industry is $n = 25$. In this example, the profit of firms in an all-green market is higher than the profit of firms in an all-brown market.

From the numerical experiments, we observe that multiple steady states are more likely to appear when the substitutability parameter S is high, indicating that consumers perceive the two types of wine as similar products.

5 Welfare considerations

In the previous sections, we showed that, depending on the model parameters, various industry configurations may arise at the steady state. We now investigate the relative desirability

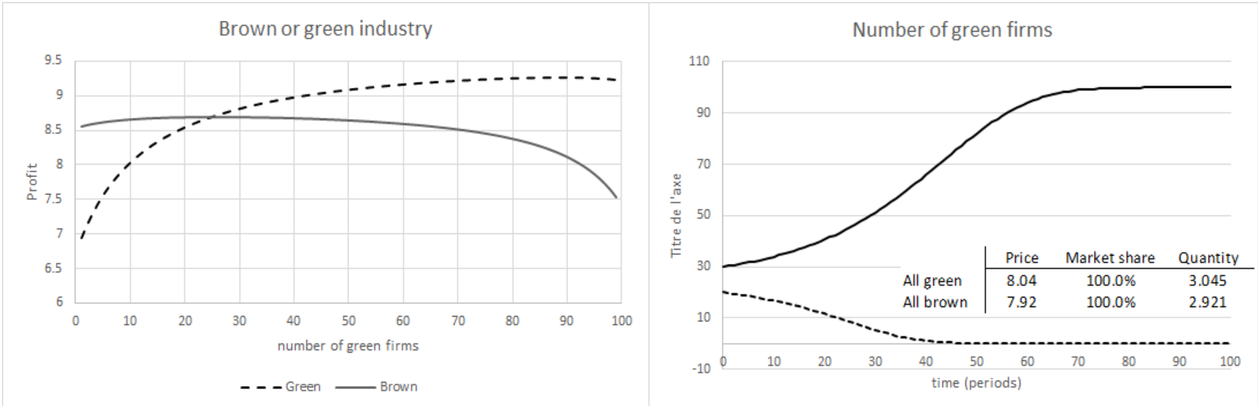

Figure 8: Efficiency E_G increasing with the number of green producers, with two coexisting steady states, $n^* = 100$ and $n^* = 0$, according to the initial conditions. Parameter values are $N = 100$, $S = 0.96$, $F_G = F_B$, $E_B = 295$, $E_N = 307.5$, $E_0 = 283$, $f_G = 0.05$. This could correspond, for instance, to an increase in A_G from 301 to 312.5 and a decrease in m_G from 18 to 5, as n varies from 0 to 100.

of long-run outcomes in terms of global welfare.

For a given industry size and composition n, the producer surplus is defined as

$$\begin{aligned} PS(n) &= PS_G + PS_B \\ &= nF_G q_G^2 + (N-n) F_B q_B^2 \end{aligned}$$

and the consumer surplus as

$$\begin{aligned} CS(n) &= U(Q_G, Q_B) - P_G Q_G - P_B Q_B \\ &= \frac{1}{2}\left(F_G (nq_G)^2 + 2Sn(N-n) q_G q_B + F_B ((N-n) q_B)^2\right). \end{aligned}$$

The total welfare is then given by

$$\begin{aligned} W(n) &= PS(n) + CS(n) \\ &= \frac{1}{2} F_B q_B^2 (N-n)(N-n+2) + \frac{1}{2} n F_G q_G^2 (n+2) \\ + & \quad Sn(N-n) q_G q_B, \; n \in (0, N) \qquad (12) \\ W(0) &= \frac{N(N+2)}{2(N+1)^2} \frac{E_B^2}{F_B} \qquad (13) \\ W(N) &= \frac{N(N+2)}{2(N+1)^2} \frac{E_N^2}{F_G} \qquad (14) \end{aligned}$$

where E_N is the maximum value of E_G.

A first question to answer is whether, and under what conditions, a green industry is more socially desirable than a brown one. By comparing the welfare in equations (13)-(14), a green industry is better than a brown industry when

$$\frac{E_N^2}{F_G} > \frac{E_{B.}^2}{F_B}. \tag{15}$$

For a green industry to be the most desirable outcome, the maximum efficiency of green firms, in terms of quality or production cost, must be large enough, compared to the efficiency of brown firms with respect to the respective slope of their inverse demand curve. This result can be directly applied to the case illustrated in Figure 8, where the two coexisting steady states are given by either a brown industry or a green industry. In this instance, condition (15) is satisfied, and a green industry is the socially desirable outcome. However, to reach this optimal industry composition, an initial group of at least 25 sustainable wine producers is needed. Condition (15) is also satisfied in the cases illustrated in Figures 4 to 7, but is not satisfied in the cases illustrated in Figures 2 and 3.

A second interesting question is understanding whether the socially optimal industry composition can be attained at the steady state. The welfare-maximizing industry composition is obtained by differentiating (12) with respect to the number of green firms and equating the resulting expression to zero. Even when the model parameters are assumed to be constant, the impact of a change in the industry composition on the total welfare a complex expression, and its roots cannot be found analytically (see Appendix 7.2). Insights can be obtained through numerical experiments.

Simulations show that, when the parameters are constant, the steady-state value of n, where the profits of the two types of firms are equal, can be either the optimal industry composition or a composition very close to it, meaning that profit-based decisions about whether or not to change production practices will lead to a close approximation of the total welfare-maximizing industry configuration.

This is what happens in Figure 2, where the steady-state brown industry is also the welfare-maximizing one. For the two "California" cases illustrated in Figures 2 and 3, the steady-state compositions of the industry are respectively 21 and 48 green firms, while the corresponding optimal-welfare compositions are 21 and 45 green firms respectively (see Figure 9). In the second case, the welfare loss at the steady state is 0.0004%.

For the two "New Zealand" illustrations (Figures 4 and 5), the steady-state composition of the industry is 98 green firms. Figure 10 shows that the welfare function reaches its maximum value when the number of green producers is 98 in the first case and 96 in the second (for a welfare loss of 0.0016% in the second case).

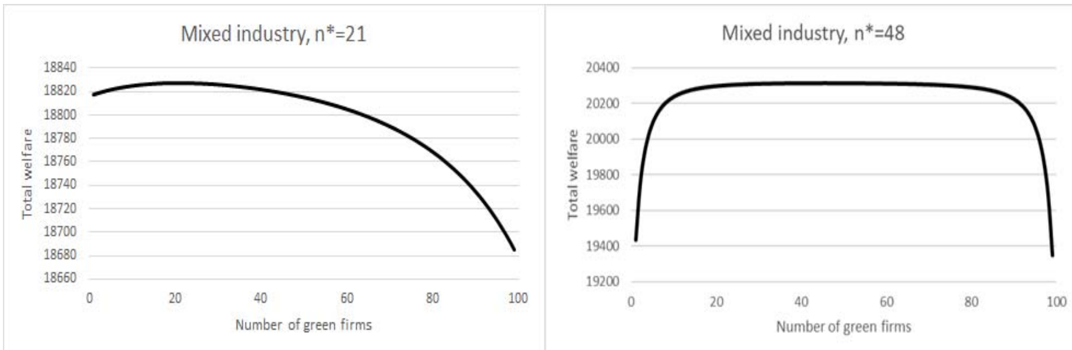

Figure 9: Total welfare as a function of the number of green firms in the industry. In the left panel, parameter values correspond to those of Figure 2, where the steady state is 21. In the right panel, parameter values correspond to those of Figure 3, where the steady state is 48.

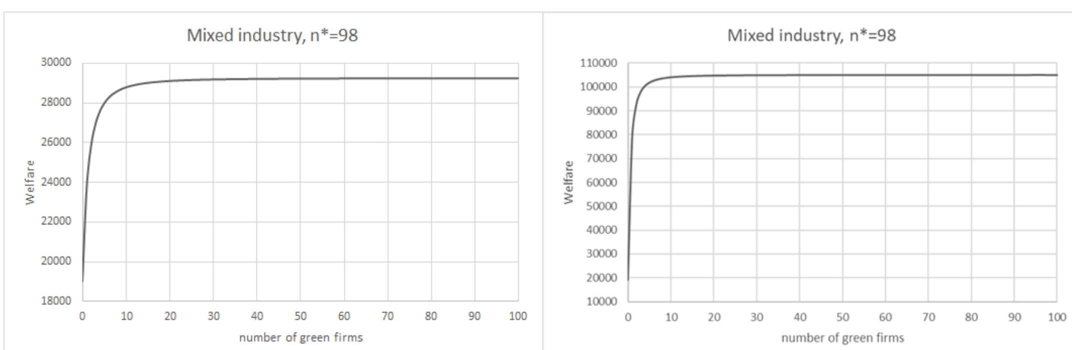

Figure 10: Total welfare as a function of the number of green firms in the industry. In the left panel, parameter values correspond to those of Figure 4. In the right panel, parameter values correspond to those of Figure 5. In both cases, the steady state is 98.

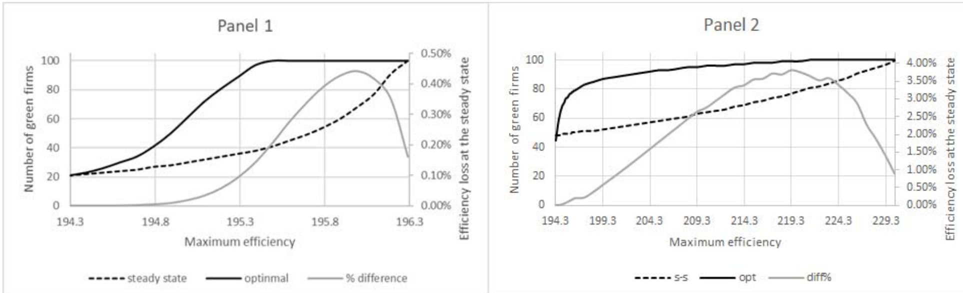

Figure 11: Impact of the maximum efficiency E_N when the efficiency of green firms is given by $E_G(n) = E_0 + \frac{EN-n}{N}$. Parameters in Panel 1 and Panel 2 are the same as in Figure 2 and Figure 3, respectively, with $E_0 = 194.29$.

When, however, E_G is increasing with n, we find that the steady-state composition of the industry can differ significantly from the socially optimal industry composition, depending on the characteristics of the industry. When the free market reaches a steady-state configuration that is far from the socially optimal one, this indicates that some potential gains from exchange are not materialized, leaving room for external intervention. We find that the maximum efficiency of the green industry (that is, the parameter $E_G(N) = E_N$) has a positive impact on the number of green firms, both at the steady state and at the socially optimal composition; however, these two numbers are not impacted in the same way as the maximum efficiency E_N increases. This is illustrated in Figure 11, where market parameters are as in Figures 2 and 3. In both cases, starting from the constant parameter case ($E_N = E_0$), we observe that the distance between the number of green firms at the welfare-maximizing configuration increases faster than the number of green firms at the steady-state composition, before stabilizing and then decreasing for large values of E_N. However, the largest difference between the number of firms at the optimal welfare composition and at the steady-state composition does not necessarily imply the largest welfare loss.

6 Conclusion

In this paper, we propose a general oligopoly model where groups of firms are characterized by different production methods, involving different fixed costs, to produce differentiated goods. This differentiated oligopoly model is used to analyze the specific case with two groups (e.g. green or brown production methods), where we assume that individual firms have the possibility of changing their production technology from one period to another, according to the relative performance of the two groups.

After characterizing the equilibrium solution of the static model and analyzing its response to the demand function parameters and to the relative weight of both groups in the industry, we identify conditions on the fixed cost leading to different long-run industry compositions.

We specialize this analysis for the wine industry, and show that our model can produce equilibrium market compositions that are presently observed in various countries where green and brown wine industries coexist. We then consider the possibility that some parameters are not constant, but depend on the number of firms producing the green variety due to a reputation or efficiency spillover effect. When this is the case, we find that multiple steady states may occur, depending on the initial size of the green industry.

Finally, we assess the possible steady-state compositions against the welfare-maximizing one. Special attention is given to the extreme cases of a completely green or completely brown industry. When the market parameters are constant, we find that, in general, the industry composition tends to stabilize near a value that optimizes the global welfare. However, when the efficiency of green firms increases with the relative weight of the green industry, the steady state achieved without intervention is no longer the first-best industry composition, so that policy intervention may be needed.

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7 Appendix

7.1 Impact of the substitutability parameter

7.1.1 Impact on individual output

The impact of S on the equilibrium quantity of a firm in group k is

$$\begin{aligned} \frac{dq_k}{dS} &= n_j \frac{-E_j n_k n_j S^2 + 2F_j E_k n_k (n_j+1) S - F_k F_j E_j (n_j+1)(n_k+1)}{\Omega^2} \\ &= n_j \frac{S n_k q_k - F_j q_j (n_j+1)}{\Omega}, \;\; k,j \in \{B,G\}, \; k \neq j. \end{aligned}$$

The numerator is a concave parabola in S with a negative intercept and discriminant

$$\Delta_k = 4F_j n_k (n_j + 1) \left(F_j E_k^2 n_k (n_j + 1) - F_k E_j^2 n_j (n_k + 1) \right), \quad k, j \in \{B, G\}, \ k \neq j.$$

When S vanishes (products are independent), both derivatives are negative. For $S > 0$, if $\frac{F_B}{E_B^2} \frac{n_B+1}{n_B} = \frac{F_G}{E_G^2} \frac{n_G+1}{n_G}$, then $\Delta_G = \Delta_B = 0$, so that both derivatives are non-positive.

Otherwise, assume w.l.g. that $\frac{F_B}{E_B^2} \frac{n_B+1}{n_B} > \frac{F_G}{E_G^2} \frac{n_G+1}{n_G}$, so that $\Delta_G > 0$ and $\Delta_B < 0$. In that case, $\frac{dq_B}{dS} < 0$ and $\frac{dq_G}{dS} \lesseqgtr 0$, depending on the value of S. The roots of the numerator of $\frac{dq_G}{dS}$ are

$$F_B \frac{E_G}{E_B} \frac{n_B + 1}{n_B} \pm \frac{\sqrt{\Delta_G}}{2 E_B n_G n_B}.$$

It is straightforward to check that

$$0 < F_B \frac{E_G}{E_B} \frac{n_B + 1}{n_B} - \frac{\sqrt{\Delta_G}}{2 E_B n_G n_B} < \frac{F_G E_B}{E_G} \frac{n_G + 1}{n_G}.$$

Note that the equilibrium solution is interior iff $S < \frac{F_G E_B}{E_G} \frac{n_G+1}{n_G} < F_B \frac{E_G}{E_B} \frac{n_B+1}{n_B}$. Consequently, both derivatives are non-positive if

$$S < F_B \frac{E_G}{E_B} \frac{n_B + 1}{n_B} - \frac{\sqrt{\Delta_G}}{2 E_B n_G n_B}$$

and $\frac{dq_G}{dS}$ is positive if

$$F_B \frac{E_G}{E_B} \frac{n_B + 1}{n_B} - \frac{\sqrt{\Delta_G}}{2 E_B n_G n_B} < S < \min \left\{ \frac{F_G E_B}{E_G} \frac{n_G + 1}{n_G}, \sqrt{F_G F_B} \right\}.$$

This is feasible if

$$F_B \frac{E_G}{E_B} \frac{n_B + 1}{n_B} - \frac{\sqrt{\Delta_G}}{2 E_B n_G n_B} < \min \left\{ \frac{F_G E_B}{E_G} \frac{n_G + 1}{n_G}, \sqrt{F_G F_B} \right\}. \tag{16}$$

It can be checked that Condition 16 is satisfied when

$$\frac{F_G E_B^2}{F_B E_G^2} < \frac{4 n_G^2 (n_B + 1)^2}{(N + 2 n_B n_G + 1)^2}. \tag{17}$$

When $\frac{F_G E_B^2}{F_B E_G^2} = 1$, this reduces to

$$n_G > n_B + 1.$$

7.1.2 Impact on the total output

The total output is

$$Q = n_G q_G + n_B q_B$$

and the impact of a change in S is given by

$$\begin{aligned}\frac{dQ}{dS} &= n_B \frac{n_G}{\Omega^2} \left(-S^2 n_B n_G \left(E_B + E_G\right) + 2S \left(F_G E_B n_B \left(n_G + 1\right) + F_B E_G n_G \left(n_B + 1\right)\right)\right. \\ &\quad \left.-F_B F_G \left(n_G + 1\right)\left(n_B + 1\right)\left(E_B + E_G\right)\right)\end{aligned}$$

The numerator is a concave parabola in S with a negative intercept and roots

$$\begin{aligned}&\frac{F_G E_B n_B \left(n_G + 1\right) + F_B E_G n_G \left(n_B + 1\right) \pm n_B n_G \sqrt{\Delta}}{n_B n_G \left(E_B + E_G\right)} \\ \Delta &= E_B E_G \left(F_B \frac{n_B + 1}{n_B} - F_G \frac{n_G + 1}{n_G}\right)\left(F_B \frac{E_G}{E_B}\frac{n_B + 1}{n_B} - F_G \frac{E_B}{E_G}\frac{n_G + 1}{n_G}\right).\end{aligned}$$

If $\Delta \leq 0$, then $\frac{dQ}{dS} \leq 0$. Otherwise, assume w.l.g. that

$$\begin{aligned}F_B \frac{n_B + 1}{n_B} &> F_G \frac{n_G + 1}{n_G} \\ \frac{F_B}{E_B^2}\frac{n_B + 1}{n_B} &> \frac{F_G}{E_G^2}\frac{n_G + 1}{n_G}.\end{aligned} \tag{18}$$

It is straightforward to check that

$$\begin{aligned}0 &< \frac{F_G E_B n_B \left(n_G + 1\right) + F_B E_G n_G \left(n_B + 1\right) - n_B n_G \sqrt{\Delta}}{n_B n_G \left(E_B + E_G\right)} < \frac{F_G E_B}{E_G}\frac{n_G + 1}{n_G} \text{ if } E_B > E_G \\ 0 &< \frac{F_G E_B}{E_G}\frac{n_G + 1}{n_G} \leq \frac{F_G E_B n_B \left(n_G + 1\right) + F_B E_G n_G \left(n_B + 1\right) - n_B n_G \sqrt{\Delta}}{n_B n_G \left(E_B + E_G\right)} \text{ if } E_B \leq E_G.\end{aligned}$$

Recall that the equilibrium solution is interior iff $S < F_G \frac{E_B}{E_G}\frac{n_G+1}{n_G} < F_B \frac{E_G}{E_B}\frac{n_B+1}{n_B}$. Consequently, the derivative $\frac{dQ}{dS} > 0$ if

$$\begin{aligned}E_B &> E_G \\ \frac{F_G E_B n_B \left(n_G + 1\right) + F_B E_G n_G \left(n_B + 1\right) - n_B n_G \sqrt{\Delta}}{n_B n_G \left(E_B + E_G\right)} &< S < \min\left\{F_G \frac{E_B}{E_G}\frac{n_G + 1}{n_G}, \sqrt{F_B F_G}\right\},\end{aligned}$$

which is feasible if

$$\frac{F_G E_B n_B \left(n_G + 1\right) + F_B E_G n_G \left(n_B + 1\right) - n_B n_G \sqrt{\Delta}}{n_B n_G \left(E_B + E_G\right)} < \min\left\{F_G \frac{E_B}{E_G}\frac{n_G + 1}{n_G}, \sqrt{F_B F_G}\right\}. \tag{19}$$

It can be checked that condition 19 is satisfied when

$$\frac{F_G E_B^2}{F_B E_G^2} < \frac{1}{n_B(n_G+1)}\left(n_G(n_B+1) - \frac{F_G(E_B+E_G)^2(N+1)^2}{4E_G^2(F_B n_G(n_B+1) - F_G n_B(n_G+1))}\right). \quad (20)$$

When $F_G = F_B = F$, Assumption (18) becomes $n_B < n_G$ and (20) reduces to

$$1 < \frac{E_B^2}{E_G^2} < \frac{n_G(n_B+1)}{n_B(n_G+1)} - \frac{(E_B+E_G)^2(N+1)^2}{4n_B(n_G+1)E_G^2(n_G(n_B+1) - n_B(n_G+1))},$$

which is impossible for $E_B > E_G$:

$$\begin{aligned} & \frac{n_G(n_B+1)}{n_B(n_G+1)} - \frac{(E_B+E_G)^2(N+1)^2}{4n_B(n_G+1)E_G^2(n_G(n_B+1) - n_B(n_G+1))} \\ = \; & 1 - \frac{(N+1)(E_B+E_G) + 2E_G(n_G-n_B)}{4E_G^2 n_B(n_G+1)(n_G-n_B)}((N+1)(E_B+E_G) - 2E_G(n_G-n_B)) \\ < \; & 1 - \frac{(N+1)(E_B+E_G) + 2E_G(n_G-n_B)}{4E_G^2 n_B(n_G+1)(n_G-n_B)}(2E_G(2n_B+1)) < 1. \end{aligned}$$

7.2 Impact of market composition and size

7.2.1 Variable N and variation in the number of firms in one group

It is straightforward to check that, for $k, j \in \{G, B\}$ and $k \neq j$,

$$\begin{aligned} \frac{dq_k}{dn_k} &= -q_k \frac{(F_B F_G - S^2)n_j + F_j F_k}{\Omega} \leq 0 \\ \frac{dq_j}{dn_k} &= -SF_k \frac{q_k}{\Omega} \leq 0, \end{aligned}$$

so that an increase in the number of firms in a given group has a negative impact on the output of all firms in the industry.

The impact of an increase in n_k on the total output is given by

$$\frac{dQ}{dn_k} = F_k q_k \frac{F_j(n_j+1) - Sn_j}{\Omega}.$$

7.2.2 Constant N and variation in the number of firms in one group

For $k, j \in \{G, B\}$ and $k \neq j$,

$$\begin{aligned}\frac{dq_k}{dn_k} &= \frac{-S^3 E_j n_j^2 + S^2 E_k F_j \left(n_j - n_k + n_j^2\right) + S E_j F_j F_k \left(N + n_j^2 + 1\right) - E_k F_j^2 F_k \left(n_j + 1\right)^2}{\Omega^2} \\ &= -\frac{dq_k}{dn_j}.\end{aligned}$$

Recall that $S \leq \min\left\{\frac{F_B E_G}{E_B}, \frac{F_G E_B}{E_G}, \sqrt{F_G F_B}\right\}$. Assume that $n_j \geq n_k$. This implies that

$$E_k F_j \left(\frac{n_j - n_k}{n_j^2} + 1\right) - S E_j \geq E_k F_j - S E_j \geq 0.$$

We then have that, for $S \leq \sqrt{F_G F_B}$,

$$\begin{aligned}\frac{dq_k}{dn_k} &= \frac{S^2 \left(E_k F_j \left(n_j - n_k + n_j^2\right) - S E_j n_j^2\right) + S E_j F_j F_k \left(N + n_j^2 + 1\right) - E_k F_j^2 F_k \left(n_j + 1\right)^2}{\Omega^2} \\ &\leq \frac{F_G F_B \left(N + 1\right)}{\Omega^2} \left(S E_j - E_k F_j\right) \leq 0 \text{ if } S \text{ is feasible.}\end{aligned}$$

We conclude that the impact of a marginal increase in the number of firms in the smallest group on the quantity they produce is never positive.

Define

$$\begin{aligned}Y_k\left(S\right) &\equiv -S^3 E_j \left(N - n_k\right)^2 + S^2 E_k F_j \left(N - 2n_k + \left(N - n_k\right)^2\right) \\ &\quad + S E_j F_j F_k \left(N + 1 + \left(N - n_k\right)^2\right) - E_k F_j^2 F_k \left(N - n_k + 1\right)^2 \\ &= \Omega^2 \frac{dq_k}{dn_k}.\end{aligned}$$

Y_k is increasing in n_k and is a third-degree polynomial of S, negative, and increasing convex at $S = 0$. Moreover, at $S = \sqrt{F_G F_B}$,

$$Y_k\left(\sqrt{F_G F_B}\right) = \left(N + 1\right) F_G F_B \left(\sqrt{F_G F_B} E_j - E_k F_j\right) \text{ for } k, j \in \{G, B\} \text{ and } k \neq j,$$

the sign of which depends on the sign of $\sqrt{F_G F_B} E_j - E_k F_j$.

i) Assume $F_B E_G^2 = F_G E_B^2$, so that $\sqrt{F_G F_B} = \frac{E_G F_B}{E_B} = \frac{E_B F_G}{E_G}$.Compute

$$\frac{dY_k}{dS} = -3S^2 E_j \left(N - n_k\right)^2 + 2S E_k F_j \left(N - 2n_k + \left(N - n_k\right)^2\right) + E_j F_j F_k \left(N + 1 + \left(N - n_k\right)^2\right).$$

At $S=\sqrt{F_G F_B}$, $Y_k\left(\sqrt{F_G F_B}\right)=0$ and $\frac{dY_k\left(\sqrt{F_G F_B}\right)}{dS}=E_j F_B F_G\left(3N-4n_k+1\right)$. If $n_k<\frac{3N+1}{4}$, $Y_k\left(S\right)<0$ for $S\in\left[0,\sqrt{F_G F_B}\right)$. If, however, $n_k>\frac{3N+1}{4}$, then Y_k is decreasing in S at $S=\sqrt{F_G F_B}$, so that $\frac{dq_k}{dn_k}>0$ when S is large enough. Note that this result applies to the case when $F_k=F_j=F$ and $E_k=E_j=E$, as in Dou & Ye (2017). In that specific case, $\frac{dq_k}{dn_k}>0$ for

$$
\begin{aligned}
n_k &> \frac{3N+1}{4} \\
n_k &> N+\frac{F}{F-S}-\sqrt{\frac{FS}{F+S}+\frac{N}{F-S}+\frac{2F}{\left(F-S\right)^2}}.
\end{aligned}
$$

ii) Assume w.l.g. that $F_B E_G^2<F_G E_B^2$, which implies that $\sqrt{F_B F_G}<\frac{F_G E_B}{E_G}$ and $\frac{F_G E_B}{E_G}>\frac{F_B E_G}{E_B}$. Assume further that $\frac{F_B E_G}{E_B}<\sqrt{F_B F_G}<\frac{F_G E_B}{E_G}$. At $S=\frac{F_B E_G}{E_B}$,

$$
\begin{aligned}
Y_G\left(\frac{F_B E_G}{E_B}\right) &= E_G F_B^2\left(E_B^2 F_G-E_G^2 F_B\right)\frac{n_G-n_B}{E_B^2}>0 \text{ for } n_G>n_B \\
Y_B\left(\frac{F_B E_G}{E_B}\right) &= -F_B\left(E_B^2 F_G-E_G^2 F_B\right)\frac{E_B^2 F_G\left(n_G+1\right)^2-E_G^2 F_B n_G^2}{E_B^3} \\
&< -F_B F_G\frac{E_B^2 F_G-E_G^2 F_B}{E_B} \\
&= -F_B F_G\frac{E_B^2 F_G-E_G^2 F_B}{E_B}<0.
\end{aligned}
$$

We conclude that $\frac{dq_G}{dn_G}>0$ when $n_G>n_B$ and S is large enough, while $\frac{dq_B}{dn_B}$ is always negative.

iii) Assume that $\sqrt{F_B F_G}<\frac{F_B E_G}{E_B}<\frac{F_G E_B}{E_G}$. At $S=\sqrt{F_B F_G}$,

$$
\begin{aligned}
Y_G\left(\sqrt{F_B F_G}\right) &= \left(N+1\right)F_G F_B\left(\sqrt{F_G F_B}E_B-E_G F_B\right)<0 \\
Y_B\left(\sqrt{F_B F_G}\right) &= \left(N+1\right)F_G F_B\left(\sqrt{F_G F_B}E_G-E_B F_G\right)<0.
\end{aligned}
$$

Since $Y_G\left(\frac{F_B E_G}{E_B}\right)>0$ at $S=\frac{F_B E_G}{E_B}>\sqrt{F_G F_B}$, we conclude that $\frac{dq_G}{dn_G}<0$ for all feasible S while $\frac{dq_B}{dn_B}\leq 0$ for $n_G\geq n_B$.

Finally,

$$
\begin{aligned}
Y_B(S) &= -S^3 E_G\left(N-n\right)^2+S^2 E_B F_G\left(N-2n+\left(N-n\right)^2\right) \\
&\quad +S E_G F_G F_B\left(N+1+\left(N-n\right)^2\right)-E_B F_G^2 F_B\left(N-n+1\right)^2 \\
&< -N S^2 E_B F_G+S E_G F_G F_B\left(N+1\right)-E_B F_G^2 F_B \\
&< 0 \text{ if } E_G^2 F_B<\frac{4N}{\left(N+1\right)^2}E_B^2 F_G<E_B^2 F_G.
\end{aligned}
$$

We conclude that when n_B is large enough and when $E_G^2 F_B \to E_B^2 F_G$, $\frac{dq_B}{dn_B}$ can take positive values for $0 < S < \sqrt{F_G F_B}$.

7.2.3 Proportional increase in both groups

Assuming that the size in both groups is multiplied by $(1+\theta)$ yields for $j,k \in \{G,B\}$ and $j \neq kj$

$$
\begin{aligned}
q_k &= \frac{F_j E_k \left((1+\theta) n_j + 1\right) - S E_j (1+\theta) n_j}{F_j F_k \left((\theta+1) N + 1\right) + n_j n_k (\theta+1)^2 Y} \\
\frac{dq_k}{d\theta} &= -\frac{F_j F_k \left(F_j E_k n_k + S E_j n_j\right) + Y n_j n_k (1+\theta) \left(2 F_j E_k + X_k n_j + \theta X_k n_j\right)}{\left(F_j F_k \left(N(\theta+1)+1\right) + Y n_j n_k (\theta+1)^2\right)^2} < 0.
\end{aligned}
$$

A proportional increase in the number of firms decreases the output of firms in both groups. It has an ambiguous impact on the total output.

7.3 Proof of proposition 1

Proof. We first show that $\lambda_2 > \lambda_1$.Assume w.l.g. that $E_B^2 F_G \leq E_G^2 F_B$, so that $S < E_B \frac{F_G}{E_G} \leq E_G \frac{F_B}{E_B}$. It comes

$$
\begin{aligned}
\lambda_2 - \lambda_1 &= \frac{\left(NSE_B - F_B E_G (N+1)\right)^2 - F_B F_G E_B^2}{F_B^2 F_G (N+1)^2} - \frac{F_B \left(F_B F_G E_G^2 - \left(NSE_G - F_G E_B (N+1)\right)^2\right)}{F_B^2 F_G^2 (N+1)^2} \\
&= N \frac{S^2 N \left(E_B^2 F_G + E_G^2 F_B\right) - 4 S E_B E_G F_B F_G (N+1) + F_B F_G \left(E_B^2 F_G + E_G^2 F_B\right)(N+2)}{F_B^2 F_G^2 (N+1)^2} \\
&= N \frac{h(S)}{F_B^2 F_G^2 (N+1)^2}
\end{aligned}
$$

where $h(S)$ is a quadratic convex function of S, minimized at

$$
\begin{aligned}
S^* &= \frac{2 F_B F_G E_B E_G}{E_B^2 F_G + E_G^2 F_B} \frac{N+1}{N} > \frac{2 F_B F_G E_B E_G}{E_B^2 F_G + E_G^2 F_B} \\
&\geq \frac{E_B F_G}{E_G}.
\end{aligned}
$$

As a consequence, for $S < \min\left\{\sqrt{F_G F_B}, E_B \frac{F_G}{E_G}, E_G \frac{F_B}{E_B}\right\}$,

$$\begin{aligned} h(S) &> \left(\frac{E_B F_G}{E_G}\right)^2 N\left(E_B^2 F_G + E_G^2 F_B\right) - 4\left(\frac{E_B F_G}{E_G}\right) E_B E_G F_B F_G (N+1) \\ &\quad + F_B F_G \left(E_B^2 F_G + E_G^2 F_B\right)(N+2) \\ &= F_G\left(E_G^2 F_B - E_B^2 F_G\right) \frac{2E_G^2 F_B + N\left(E_G^2 F_B - E_B^2 F_G\right)}{E_G^2} \geq 0, \end{aligned}$$

and $\lambda_2 > \lambda_1$.

The stability condition (9) can be rewritten as

$$L(n^*) = R(n^*) \tag{21}$$

where

$$L(n) = F_G\left(F_B E_G (N-n+1) - S E_B (N-n)\right)^2 - F_B\left(F_G E_B (n+1) - S E_G n\right)^2 \tag{22}$$

$$R(n) = \delta\left(F_G F_B (n+1)(N-n+1) - S^2 n (N-n)\right)^2. \tag{23}$$

The function $R(n)$ is a concave, positive, fourth-degree polynomial, symmetric w.r.t. n, with $R(0) = R(N) = \delta F_B^2 F_G^2 (N+1)^2 \geq 0$. The function $L(n)$ is a quadratic function of n, with

$$\begin{aligned} L'(n) &= 2n\left(E_G^2 F_B - E_B^2 F_G\right)\left(F_B F_G - S^2\right) \\ &\quad - 2F_G\left(E_G^2 F_B^2 (N+1) - 2S E_B E_G F_B (N+1) + E_B^2\left(N S^2 + F_B F_G\right)\right). \end{aligned}$$

Note that $L'(n)$ is increasing in n if $E_G^2 F_B > E_B^2 F_G$ and decreasing if $E_G^2 F_B < E_B^2 F_G$. We now show that L is a strictly decreasing function of n.

1. First case: $E_G^2 F_B > E_B^2 F_G$, which implies that $S < \min\left\{\sqrt{F_G F_B}, E_B \frac{F_G}{E_G}\right\}$. We then have

$$\begin{aligned} L'(n) &\leq L'(N) \\ &= -2S^2 N E_G^2 F_B + 4S E_B E_G F_B F_G (N+1) - 2F_B F_G\left(E_B^2 F_G (N+1) + E_G^2 F_B\right). \end{aligned}$$

   This expression is concave in S and maximized at $S^* = \frac{E_B F_G}{E_G} \frac{N+1}{N} > \frac{E_B F_G}{E_G}$.We then have

$$
\begin{aligned}
L'(n) &< -2\left(\frac{E_B F_G}{E_G}\right)^2 N E_G^2 F_B + 4\left(\frac{E_B F_G}{E_G}\right) E_B E_G F_B F_G (N+1) \\
&\quad -2F_B F_G \left(E_B^2 F_G (N+1) + E_G^2 F_B\right) \\
&= 2F_B F_G \left(E_B^2 F_G - E_G^2 F_B\right) < 0,
\end{aligned}
$$

and L is a strictly decreasing convex function of n.

2. Second case: $E_G^2 F_B \leq E_B^2 F_G$, which implies that $S < \min\left\{\sqrt{F_G F_B}, E_G \frac{F_B}{E_B}\right\}$. We then have

$$
\begin{aligned}
L'(n) &\leq L'(0) \\
&= -2NS^2 E_B^2 F_G + 4S E_B E_G F_B F_G (N+1) - 2F_G F_B \left(E_B^2 F_G + E_G^2 F_B (N+1)\right)
\end{aligned}
$$

This expression is concave in S and maximized at $S^* = \frac{E_G F_B}{E_B} \frac{N+1}{N} > \frac{E_G F_B}{E_B}$. We then have

$$
\begin{aligned}
L'(n) &< -2N\left(\frac{E_G F_B}{E_B}\right)^2 E_B^2 F_G + 4\left(\frac{E_G F_B}{E_B}\right) E_B E_G F_B F_G (N+1) \\
&\quad -2F_G F_B \left(E_B^2 F_G + E_G^2 F_B (N+1)\right) \\
&= -2F_B F_G \left(E_B^2 F_G - E_G^2 F_B\right) \leq 0
\end{aligned}
$$

and L is a strictly decreasing concave function of n.

We have shown that L is a strictly decreasing function of n, which implies that $L(0) > L(N)$. The steady state is defined by the intersection of a strictly decreasing (concave or convex) function and a concave symmetric function. We distinguish three cases.

1. $L(0) \leq R(0)$, which corresponds to $\delta \geq \lambda_2$.

   Since $L(n) < L(0) \leq R(0) < R(n)$ for all $n > 0$, **there are only brown firms in the industry in the steady state.**

2. $L(0) > R(0)$, which corresponds to $\delta < \lambda_2$, and $L(N) < R(N)$, which corresponds to $\delta > \lambda_1$.

   Since $L(N) < R(N) \leq R(n)$ for all $n \in [0, N]$ and $L(0) > R(0)$, there exists at least one n^* where $L(n^*) = R(n^*)$. On the other hand, since L is convex, linear, or concave, while R is concave, there exist at most two intersection points between the two curves.

Since $L(N) < R(N)$, a second intersection point cannot be in $(n^*, N]$, and **there exists a single steady state where green and brown firms coexist.**

3. Otherwise, $L(N) \geq R(N)$, which corresponds to $\delta \leq \lambda_1$. For this third case to happen, we need $\lambda_1 \geq 0$:

$$\begin{aligned} & F_B F_G E_G^2 - (NSE_G - F_G E_B (N+1))^2 \\ = \; & -N^2 S^2 E_G^2 + 2N F_G E_B E_G (N+1) S + F_B F_G E_G^2 - F_G^2 E_B^2 (N+1)^2 \geq 0, \end{aligned}$$

or, equivalently,

$$S \in \left[\frac{E_B F_G}{E_G} \frac{N+1}{N} - \frac{\sqrt{F_B F_G}}{N}, \min \left\{ \sqrt{F_G F_B}, E_G \frac{F_B}{E_B}, E_B \frac{F_G}{E_G} \right\} \right),$$

which requires

$$E_B^2 F_G < E_G^2 F_B.$$

Notice that λ_1 is increasing in S over the interval of feasible values for S.

Define $\beta(n) \equiv L(n) - R(n)$.We will show that $\beta(n) > 0$ for all $n \in [0, N]$.

Compute $\beta''(n)$:

$$\begin{aligned} \beta''(n) = \; & 2 \left(F_B F_G - S^2 \right) \left(E_G^2 F_B - E_B^2 F_G + \delta \left(N^2 S^2 - F_B F_G (N(N-2) - 2) \right) \right. \\ & \left. + 6\delta n \left(F_B F_G - S^2 \right) (N - n) \right) \\ \geq \; & 2 \left(F_B F_G - S^2 \right) \left(E_G^2 F_B - E_B^2 F_G + \delta \left(S^2 N^2 - F_B F_G (N(N-2) - 2) \right) \right). \end{aligned}$$

If $S^2 N^2 - F_B F_G (N(N-2) - 2) \geq 0$, then $\beta''(n) > 0$. Otherwise, using $\delta \leq \lambda_1$,

$$\begin{aligned} & \delta \left(S^2 N^2 - F_B F_G (N(N-2) - 2) \right) \\ \geq \; & \frac{F_B F_G E_G^2 - (NSE_G - F_G E_B (N+1))^2}{F_B F_G^2 (N+1)^2} \left(N^2 S^2 - F_B F_G (N(N-2) - 2) \right). \end{aligned}$$

Note that since λ_1 is increasing with S, $\lambda_1 (N^2 S^2 - F_B F_G (N(N-2) - 2))$ is decreasing with S. Using $S < E_B \frac{F_G}{E_G}$ we get

$$\delta \left(S^2 N^2 - F_B F_G (N(N-2) - 2) \right) > \left(N E_B^2 F_G - E_G^2 F_B (N(N-2) - 2) \right) \frac{E_G^2 F_B - E_B^2 F_G}{F_B E_G^2 (N+1)^2}$$

and finally

$$\begin{aligned}\beta''(n) &\geq 2\left(F_BF_G-S^2\right)\left(E_G^2F_B-E_B^2F_G\right.\\&\quad\left.+\left(NE_B^2F_G-E_G^2F_B\left(N\left(N-2\right)-2\right)\right)\frac{E_G^2F_B-E_B^2F_G}{F_BE_G^2\left(N+1\right)^2}\right)\\&= 2\left(F_BF_G-S^2\right)\left(E_G^2F_B-E_B^2F_G\right)\frac{E_G^2F_B\left(4N+3\right)+NE_B^2F_G}{E_G^2F_B\left(N+1\right)^2}>0.\end{aligned}$$

We have shown that β is convex over $[0,N]$, so that $\beta'(n)\leq\beta'(N)$ for all n.

Now compute $\beta'(N)$:

$$\begin{aligned}\beta'(N) &= 2\delta NF_BF_G\left(F_BF_G-S^2\right)\left(N+1\right)\\&\quad+\left(4SE_BE_GF_BF_G\left(N+1\right)-2NS^2E_G^2F_B-2F_BF_G\left(F_G\left(N+1\right)E_B^2+F_BE_G^2\right)\right)\\&< 2\frac{F_BF_GE_G^2-\left(NSE_G-F_GE_B\left(N+1\right)\right)^2}{F_BF_G^2\left(N+1\right)^2}NF_BF_G\left(F_BF_G-S^2\right)\left(N+1\right)\\&\quad+\left(4SE_BE_GF_BF_G\left(N+1\right)-2NS^2E_G^2F_B-2F_BF_G\left(F_G\left(N+1\right)E_B^2+F_BE_G^2\right)\right)\\&= -N^3\left(E_BF_G-SE_G\right)^2\left(F_BF_G-S^2\right)\\&\quad-N^2F_G\left(E_BF_G-SE_G\right)\left(2E_B\left(F_BF_G-S^2\right)+F_B\left(E_BF_G-SE_G\right)\right)\\&\quad-NF_G\left(S^2\left(E_G^2F_B-E_B^2F_G\right)+F_B\left(E_BF_G-SE_G\right)\left(3E_BF_G-SE_G\right)\right)\\&\quad-F_BF_G^2\left(E_B\left(E_BF_G-SE_G\right)+E_G\left(E_GF_B-SE_B\right)\right)\\&< 0.\end{aligned}$$

We have shown that $\beta'(n)\leq\beta'(N)<0$ for $n\in[0,N]$, which means that β is strictly decreasing. We then have $L(n)>R(n)$ for all $n\in[0,N)$, and **there are only green firms in the industry in the steady state.**

■

7.4 Comparative statics

7.4.1 Sensitivity of λ_1 and λ_2 to the model's parameters

Compute

$$
\begin{aligned}
\frac{d\lambda_1}{dF_B} &= \left(\frac{F_G E_B (N+1) - NSE_G}{F_B F_G (N+1)}\right)^2 > 0; \\
\frac{d\lambda_1}{dF_G} &= E_G \frac{2NS\left(NSE_G - F_G E_B (N+1)\right) - F_B F_G E_G}{F_B F_G^3 (N+1)^2} \\
&< -\frac{E_G}{F_G^2} \frac{F_B E_G + 2NSE_B}{F_B (N+1)^2} < 0; \\
\frac{d\lambda_1}{dE_B} &= -\frac{2\left(F_G E_B (N+1) - NSE_G\right)}{F_B F_G (N+1)} \\
&< -2\frac{E_B}{F_B (N+1)} < 0; \\
\frac{d\lambda_1}{dE_G} &= \frac{2NS\left(F_G E_B (N+1) - NSE_G\right) + 2F_B F_G E_G}{F_B F_G^2 (N+1)^2} \\
&> \frac{2}{F_G} \frac{F_B E_G + NSE_B}{F_B (N+1)^2} > 0; \\
\frac{d\lambda_1}{dS} &= 2\frac{N}{F_B F_G^2} \frac{E_G \left(F_G E_B (N+1) - NSE_G\right)}{(N+1)^2} \\
&> 2N\frac{E_G}{F_G} \frac{E_B}{F_B (N+1)^2} > 0.
\end{aligned}
$$

Similarly for λ_2 we get

$$
\begin{aligned}
\frac{d\lambda_2}{dF_B} &= E_B \frac{2NS\left(F_B E_G (N+1) - NSE_B\right) + F_B F_G E_B}{F_B^3 F_G (N+1)^2} \\
&> E_B \frac{F_G E_B + 2NSE_G}{F_B^2 F_G (N+1)^2} > 0; \\
\frac{d\lambda_2}{dF_G} &= -\left(\frac{F_B E_G - NSE_B + NF_B E_G}{F_B F_G (N+1)}\right)^2 < 0; \\
\frac{d\lambda_2}{dE_B} &= -\frac{2NS\left(F_B E_G (N+1) - NSE_B\right) + 2F_B F_G E_B}{F_B^2 F_G (N+1)^2} \\
&< -\frac{2}{F_B} \frac{F_G E_B + NSE_G}{F_G (N+1)^2} < 0; \\
\frac{d\lambda_2}{dE_G} &= \frac{2\left(F_B E_G (N+1) - NSE_B\right)}{F_B F_G (N+1)} \\
&> 2\frac{E_G}{F_G (N+1)} > 0; \\
\frac{d\lambda_2}{dS} &= -2\frac{N}{F_B^2 F_G} \frac{E_B\left(F_B E_G (N+1) - NSE_B\right)}{(N+1)^2} \\
&< -2N\frac{E_B}{F_B} \frac{E_G}{F_G (N+1)^2} < 0.
\end{aligned}
$$

7.4.2 Effect of changes in parameters on a mixed industry

Recall the functions

$$
\begin{aligned}
L(n) &= F_G\left(F_B E_G (N-n+1) - SE_B (N-n)\right)^2 - F_B\left(F_G E_B (n+1) - SE_G n\right)^2 \\
R(n) &= \delta\left(F_G F_B (n+1)(N-n+1) - S^2 n (N-n)\right)^2
\end{aligned}
$$

used to characterize a steady state where profits are equal for both types of producers.

Note that a change in $E_k = A_k - m_k$ has no impact on the function R, while

$$
\begin{aligned}
\frac{dL}{dE_B} &= 2S^2 F_G E_B (N-n)^2 - 2SF_G F_B E_G (N+1)(N-2n) - 2F_G^2 F_B E_B (n+1)^2 \\
&< 2SF_B \frac{E_G}{E_B} F_G E_B (N-n)^2 - 2SF_G F_B E_G (N+1)(N-2n) - 2F_G^2 F_B E_B (n+1)^2 \\
&= -2SF_B F_G E_G (N-n) + 2nSF_B F_G E_G (n+1) - 2F_G^2 F_B E_B (n+1)^2 \\
&< -2SF_B F_G E_G (N-n) + 2nF_G \frac{E_B}{E_G} F_B F_G E_G (n+1) - 2F_G^2 F_B E_B (n+1)^2 \\
&= -2SF_B F_G E_G (N-n) - 2F_B F_G^2 E_B (n+1) < 0.
\end{aligned}
$$

This means that a reduction in E_B (that is, either an decrease in A_B or an increase in m_B) shifts the $L(n)$ function upward for all n and has no effect on $R(n)$; as a result, the intersection of the two functions happens at a greater value of n.

In the same way, $\frac{dL}{dE_G} > 0$, so that an increase in the value of E_G (that is, either an increase in A_G or a decrease in m_G) results in a lower value of n at the steady state.

On the other hand, a change in the value of δ has no impact on function L, while

$$\frac{dR}{d\delta} = \left(S^2 n (N-n) - F_B F_G (n+1)(N-n+1)\right)^2 > 0.$$

This means that a reduction in δ shifts the $R(n)$ function downward for all n and has no effect on $L(n)$; as a result, the intersection of the two functions happens at a greater value of n.

7.5 Impact on welfare

$$\begin{aligned}\frac{dW(n)}{dn} &= F_G q_G \left((n+1) q_G + n \left((n+2)\frac{dq_G}{dn} + \frac{S(N-n)}{F_G}\frac{dq_B}{dn}\right)\right) \\ &\quad + F_B q_B \left(-(N-n+1) q_B + (N-n)\left((N-n+2)\frac{dq_B}{dn} + \frac{Sn}{F_B}\frac{dq_G}{dn}\right)\right) \\ &\quad - nSq_B q_G,\end{aligned}$$

where

$$\begin{aligned}\frac{dq_B}{dn} &= -\frac{-YX_B (N-n)^2 - F_G F_B X_B (N+1) - YF_G E_B (N-2n)}{(Yn(N-n) + F_G F_B (N+1))^2} \\ \frac{dq_G}{dn} &= \frac{-YX_G (N-n)^2 - F_B F_G X_G (N+1) - YF_B E_G (N-2n)}{(Yn(N-n) + F_G F_B (N+1))^2}.\end{aligned}$$