The Role of Grandfathering under Simultaneous Market Power in Product and Emission Permit Markets

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ABSTRACT

We analyze the behavior of polluting oligopolistic firms that interact strategically both in the output and the emissions permit markets. We consider a model of two firms competing under different oligopolistic structures while one of them is a dominant firm in the permit market. In this framework, we study and compare the effect of market power and the role of grandfathering in three different situations. We determine that the follower could always make higher profits than the Stackelberg leader if it receives enough more free permits. Under Cournot structure the firm who receives more free permits produces more and makes higher profits.

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Key words: Carbon price, Market power, Duopoly, Cournot Model, Stackelberg model

1. Introduction

A coordinated and effective global agreement to cut down greenhouse gas emissions is far from being reached after many years of formal negotiations since the first big attempt in Kyoto 1998. Meanwhile, some market-based climate change policies have been put in place at regional levels, the European Trading Scheme (ETS) being the most important experience.[1]

The efficiency of emission permit markets had deserved academic attention well before its practical implementation.[2] The role of market power and the method to do the initial allocation of permits (mainly grandfathering or auctioning) are among the main issues that have attracted the attention of researchers regarding the efficiency of this kind of markets. Montgomery (1972) established that, under perfect competition, the equilibrium allocation is cost-effective regardless the allocation method. Hahn (1984) considered market power in the permit market for the first time and showed that the market equilibrium allocation fails to be efficient in the presence of a dominant firm, because such a firm has an incentive to manipulate the permit prices in order to increase its profits. As Hahn noted, if there is a dominant firm operating in the permit market, the resulting allocation is efficient only if the dominant firm is initially allocated with the same amount of permits that it would get in a competitive equilibrium.

In its basic version, both results (Montgomery's result about the efficiency of a competitive market and Hahn's result about the lack of efficiency of a non-competitive market) focus on the permit market itself and abstract from the existence of an

[1] Although, at the time of writing this paper it was still not working in practice, the Chinese emissions market, starting in 2020, is expected to be the largest one in the world.

[2] See Baumol and Oates (1971) and Montgomery (1972) as leading examples of academic studies. Early forms of emissions trading took place in USA from 1976 based on emissions reduction credits, but it is the Acid Rain Program, enacted in 1990 and aimed to control SO_2 and NOx emissions, the one commonly considered as the first well-established market of emissions. The first phase of this program begun in 1995 (see, for example, Tietenberg 2006).

associated pollutant product market and its corresponding structure. As it has been later acknowledged by several authors, the product market must be taken into account to make a fully-fledged analysis, since the cost associated to the purchase of permits impacts the overall profits that firms aim to maximize, and therefore, a profit-maximizing firm should act in the permit market in accordance with its product market strategy. Misiolek & Elder (1989) extended Hahn's setting to the product market and concluded that a single dominant firm can manipulate permit prices to drive up the fringe firm's cost in the product market. Hinterman (2011) found that the threshold of free allocation above which a dominant firm will have an incentive to set the permit price above its marginal abatement costs is below its optimal emissions in a competitive market, and that overall efficiency cannot be achieved by means of the permit allocation alone. Hintermann (2017) pointed out that a dominant net permit buyer may want to increase the permit price, provided that the increase in compliance costs is more than offset by the sum of the revenue increase in the output market and the increase in rents embedded in free allocation.

We aim to address situations in which there is a strong interaction between the strategic behavior in the product and the permit markets. The NOx permit market in California is an important practical example. Nearly 25% of the NOx permits were allocated to facilities that sell power into the California electricity market, which has been recognized for its (unilateral) market power problems. In fact, Kolstad and Wolak (2003) argue that electric utilities used the NOx market to enhance their ability to exercise (unilateral) market power in the electricity market[3].

The connections between the permit market and the output market are very important, not only at the firm level, but also at the country level. Montero (2009)

[3] See Fowlie (2010) for the analysis of the effects on permit market efficiency.

argues that some of the large countries in an eventual global carbon market are also big players in energy markets. In this respect, Hagem and Maestad (2006) analyse the optimal strategies of Russia, which is a very relevant player in the international market for emission permits but is also an important fuel exporter. Using numerical simulations, they concluded that Russia could have benefited from coordinating its permit exports with its oil and gas exports during the commitment period of the Kyoto Protocol.

In this paper we analyze the behavior of polluting oligopolistic firms (or countries) [4] that interact strategically both in the output and the permit markets, paying particular attention to the market power position of each firm in each market. To this aim we consider a model of two firms competing under different oligopolistic structures while there is a dominant firm in the permit market. In this framework, we study and compare the effect of market power under three different situations, depending on the role that each firm plays in the product market and, specifically, if the firm that is dominant in the permit market plays the same role in the product market.

We first set up a simple model of a permit market assuming that there is a dominant firm and another one that behaves as a price taker. We show that the equilibrium of such a market is crucially determined, first, by the equilibrium in the output market and, second, by the initial allocation of permits. We also restate the classical result of Hahn (1984), which implies that, as long as there is any permit trade taking place in the market, the dominant firm is taking an advantage of its position whether it acts as a net buyer or as a net seller of permits. Then, we move on to study the output market and its link with the permit market.

[4] Although our model could be interpreted in terms of firms or in terms of countries, for simplicity of exposition, most of the time we will simply refer to "firms".

In order to investigate the role of the product market with some detail, we consider three alternative market structures. In the first version we consider a Cournot oligopoly, in which both firms play a symmetric role, whereas in the other two there are a leader and a follower a la Stackelberg. Specifically, in the second version (Stackelberg 1) we consider a dominant firm in the emission permit market that is also a leader in the product market. In the third version (Stackelberg 2) we consider that the firm that acts as a leader in the output market (Firm 1) is the follower in the permits market.

The aim of the first output model (Cournot) is to study a case in which a firm has a dominant position in the permit market but the firms are otherwise symmetric. This allows us to ask to what extent the leading position in the permit market can break the symmetry in the output market or, the other way around, how the symmetric position in the output market can soften the leadership position of one of the firms in the permit market.

The second version (Stackelberg 1) depicts a situation in which a single firm has the ability to lead both markets at the same time, which a priori is the most favorable situation for the leading firm itself but also the less favorable for the following firm and, arguably, also for the sake of market efficiency. Similar versions of this model have been addressed in the literature in the form of a Stackelberg leader-follower setting or a leader-fringe structure. Apart from Hinterman (2011), which has been mentioned above, Chevalier (2008) considers a permit market with both spatial and intertemporal trading, a big dominant firm and a large number of small firms who are nonstrategic but forward looking. The equilibrium is characterized for the monopoly case and for intermediate cases. Tanaka & Chen (2012) consider a Cournot-fringe model with market power in both product and permits market to simulate the California electricity market and they show that Cournot firms can significantly raise both power price and permit price, which results in a great loss in social surplus.

As far as we know, our third case, (Stackelberg 2) has not been analyzed in the literature yet. The motivation for this version is to look for conditions under which being a leader in one of these markets represents a competitive advantage or, in other words, which is the best place to exercise market power. With this version of the model, we try to get some insight about the relative importance of having a leadership position in the output market and in the permit market. As a motivation for this approach consider the role of Russia and USA in the Kyoto Protocol. It has been argued that USA's rejection to enter this Protocol resulted from the fact that its least costly way to implement the required targets would have involved large purchases of emission credits from Russia (see Bernard *et.al*, 2003). This situation can be captured in a simple stylized way by stating that Russia was (or the USA thought it was) a leader in the permit maker. Simultaneously, it can be argued that USA has a stronger leadership position than Russia in the output market. One can be interested in assessing the relative strength of both leaderships when it comes to get a better economic outcome. With our third model we aim to address this question in a simple and tractable way.

The remainder of this article has the following structure: In Section 2 we describe the basic elements of the model. Section 3 analyses the equilibrium in the emissions permit market. In the next three sections we explore the implications of grandfathering in the firms output and profit within three different oligopolistic structures: Cournot (Section 4) and the two versions of the Stackelberg model that we have discussed above (Sections 5 and 6). Section 7 compares all three models in terms of aggregate output and welfare. Section 8 states our conclusions.

2. The Model. Basic Elements

We consider an oligopoly model. For the sake of simplicity, we consider only two firms, indexed by $i = 1, 2$, that interact in the output and the emission permit market. Both firms enjoy an initial firm-specific free allocation of permits, S_i ($i = 1, 2$), such that the total number of permits, S, is given and each firm receives a given proportion, α and $1-\alpha$ respectively, so that $S_1 = \alpha S$ and $S_2 = (1-\alpha)S$. Denote individual output as x_i, total output as X and the inverse demand function as $P(X) = 1 - X$. We normalize the production cost to zero so that the only cost for firms is the abatement cost. Gross emissions are assumed to be proportional to the firms' output where one unit of output generates one unit of emissions. The firms can reduce emissions by either reducing output or making some abatement effort. Abated quantities are denoted by q_i ($i = 1, 2$) and net emissions, e_i, are given by gross emissions minus abatement, i.e., $e_i = x_i - q_i$. We assume the same abatement technology for both firms with abatement cost given by the quadratic function $C(q_i) = \frac{q_i^2}{2}$.[5] Regarding permit trading, we denote permits purchased by firm i as y_i if it has positive value. A negative value $y_i < 0$ means that firm i sells rather than buys permits. The market equilibrium condition implies $y_1 = -y_2$.

We also assume that the emissions generated by both firms can be perfectly monitored without cost by the regulatory authorities and firms cannot emit more than the number of permits they hold.

[5] Basically the same functions and notation are used, e.g. by De Feo (2013).

The timing of the game is the following: The first stage is the output game and the second stage is the emissions game. In the output game we consider three versions. In the first one (Cournot), both firms decide simultaneously and thus the first stage is one shot. In the second and the third versions (Stackelberg 1 and 2), the first stage has two sub-stages: the output leader moves in the first sub-stage and then the output follower moves in the second. The emissions game always has two sub-stages: in the first one, the firm that dominates the permit market sets the price (or, equivalently, its own demand for permits) and the follower decides its demand in the second sub-stage taking the permit price as given. All the versions are solved by backwards induction. Thus, we start analyzing the emissions game.

3. Permit market equilibrium with a dominant firm

As explained above, the emissions game has two sub-stages. In the last one, firm 2 choses its emissions (or, equivalently, abatement) by solving the following problem:

$$\begin{aligned} &\underset{\{q_2\}}{Min}\ \frac{q_2^2}{2} + py_2 \\ &s.t. \quad S_2 + q_2 + y_2 = x_2. \end{aligned} \tag{1}$$

We can substitute the constraint into the objective function and arrive at the familiar first-order condition, which states that marginal abatement cost equals the permit price. By solving such condition for q_2 we obtain firm 2’s demand for abatement and combining this expression with the constraint we get the net demand for permits of firm 2 as a function of the permit price:

$$q_2(p) = p, \tag{2}$$

$$y_2 = x_2 - p - S_2\,. \tag{3}$$

Moving one stage backwards, the dominant firm minimizes its own costs anticipating the reaction of the follower and taking into consideration the permit market-clearing condition given by

$$y_1 = x_1 - q_1 - S_1 = -x_2 + q_2(p) + S_2 = -y_2 . \quad (4)$$

Solving the cost-minimizing problem yields the optimal abatement of the dominant firm (details can be found in the Appendix):

$$q_1 = \frac{(2x_1 + x_2) - (1+\alpha)S}{3} . \quad (5)$$

Combining (2), (3), (4) and (5) we obtain firm 2´s optimal abatement and demand for permits as a function of output and the initial allocation of permits:

$$q_2 = \frac{(2x_2 + x_1) - (2-\alpha)S}{3} , \quad (6)$$

$$y_2 = -y_1 = \frac{(2\alpha - 1)S - (x_1 - x_2)}{3} = \frac{(S_1 - S_2) - (x_1 - x_2)}{3} = q_2 - q_1 . \quad (7)$$

Equation (7) shows that the dominant firm will be a net buyer of permits ($y_1 > 0$) when its abatement exceed the follower´s. In such a case, the dominant firm buys permits at a price that is lower than its marginal cost of abatement, as can be easily proved using equations (2) and (5):

$$p = \underbrace{q_2}_{MAC_2} < \underbrace{q_1}_{MAC_1} .$$

In the same way it can be stated that if the dominant firm is a net seller of permits the price will exceed its marginal cost of abatement. In both cases the dominant position is an instrument that firm 1 can use to reduce its cost and increase its profit. This conclusion is in the line of the classical result by Hahn (1984).

Equation (7) also reveals that the equilibrium in the permit market is driven by two main elements: first, the equilibrium in the output market, and more specifically, the difference in both firms' output, and second, the difference in the initial permit allocation. If we consider the particular case where both firms initially receive the same amount of permits, Equation (7) shows that the net demand for permits is proportional to the difference in output which means that the firm that produces more (less) output acts as a net buyer (seller) of permits. If, apart from receiving the same amount permits, both firms produced the same output, in equilibrium both firms would abate the same amount of emissions and there would not be any trade of permits. In the next section we show that this is the case under Cournot competition.

To have a full picture, in the following sections we investigate the equilibrium of the output market and how such equilibrium is influenced by the initial allocation. We consider two possibilities: first, both firms compete simultaneously in output a la Cournot, and second, there is a follower and a leader, a la Stackelberg. In the second case, in turn, we consider two possibilities depending on whether the leader in the output market is the same firm that has a similar position in the permit market or not.

4. Cournot competition in the output market

In this section we assume that, in the first stage of the game, both firms choose their output simultaneously maximizing their individual profit *a la* Cournot and anticipating that in the emissions permit market firm 1 is a dominant firm while firm 2 acts as a price taker. The profit maximization problem of firm 2 is:

$$\underset{\{x_2\}}{Max}\left[1-\left(x_1+x_2\right)\right]x_2-\frac{q_2^2}{2}-p\left(x_2-q_2-S_2\right) \quad s.t. \quad Eq \quad (2)\ and\ (6) \tag{8}$$

and, from the first order condition, we derive the reaction function of firm 2, which is given by

$$x_2 = \frac{9 + (8 - 7\alpha) S - 10x_1}{26}. \tag{9}$$

In a similar way, we obtain firm 1's reaction curve:

$$x_1 = \frac{3 + (1 + \alpha) S - 4x_2}{8}. \tag{10}$$

Solving the system given by equations (9) and (10) we obtain the optimal output of both agents in terms of the parameters of the model:

$$x_1 = \frac{7 + (9\alpha - 1) S}{28}. \tag{11}$$

$$x_2 = \frac{7 + (9 - 11\alpha) S}{28}, \tag{12}$$

Since the reaction functions are not equal, output is not necessarily equal for both firms in equilibrium, as it would be the case in the standard Cournot model with symmetric firms. This is due to the different role that the firms play in the emissions game. To have a more accurate understanding of the difference between both firms' output, we combine (11) and (12) to get

$$x_1 - x_2 = \frac{5(2\alpha - 1) S}{14} = \frac{5(S_1 - S_2)}{14}. \tag{13}$$

Plugging equations (11) and (12) into equation (5) and (6) and rearranging we obtain the optimal abatement of both firms in terms of the parameters of the model:

$$q_1 = \frac{1 - (1 + \alpha) S}{4}, \tag{14}$$

$$q_2 = \frac{7 + (5\alpha - 13) S}{28}, \tag{15}$$

and combining both expressions, we can compute the difference in the abatement made by both firms, which in turn, according to (7), provides the amount of permits traded in the market equilibrium:

$$y_1 = q_1 - q_2 = \frac{3(1-2\alpha)S}{14} = \frac{3(S_2 - S_1)}{14}. \tag{16}$$

These results may seem surprising to some extent. If both firms initially receive the same amount of permits (i.e., $S_1 = S_2$ or $\alpha = 0.5$), firm 1´s dominant position in the emission permits market does not lead to any advantage in practical terms. In fact, according to equation (13) both firms will produce the same amount of output $(x_1 = x_2)$ and, according to (16), both firms would also abate the same amount. As a consequence, there will not be any permit trade and both firms will also make the same profit (see Proposition 1 below).

Thus, the dominant position of firm 1 only plays a relevant role when the permit allocation is not symmetric and it seems natural to ask how each firm will react to a change in its permit allocation. From equations (11) and (12) it can be easily proved that firm 2's output is more sensitive to its permit allocation than firm 1's, i.e, when firm 2's share of permits increases, its output increase is higher than the firm 1´s output increase when this firm receives one more permit:

$$\frac{\partial x_2}{\partial(1-\alpha)} = -\frac{\partial x_2}{\partial\alpha} = \frac{11\alpha}{28} > \frac{9\alpha}{28} = \frac{\partial x_1}{\partial\alpha}. \tag{17}$$

On the other hand, when firm 1' share of permits increases, the firm reacts by decreasing abatement in a higher amount than firm 2 in the same situation. Formally

$$\frac{\partial q_1}{\partial\alpha} = \frac{-S}{4} < -\frac{5S}{28} = \frac{\partial q_2}{\partial(1-\alpha)}. \tag{18}$$

Summing up, the firms take different strategies due to the different role they play in the permit market. When endowed with more permits, both firms would react by increasing output and decreasing abatement, but the permit leader would decrease abatement more and increase output less than the follower does in the same circumstances. The reason lies in the strategic behavior of firm 1 as it is able to buy permits at a price lower than its marginal abatement cost, taking advantage of its leadership position.

This feature has a straightforward implication on total output, given by

$$X = \frac{7 + (4 - \alpha) S}{14}, \tag{19}$$

which is decreasing in α, i.e., the more permits are given to the dominant firm (given a fixed overall amount of permits), the less output is produced.

The equilibrium price of permits is given by

$$p = \frac{7 + (5\alpha - 13) S}{28} \tag{20}$$

which is increasing in α because, the more permits are allocated to the dominant firm, the more it will exert its market power to increase the price.

We can summarize the main results regarding firms' behavior when they receive a different amount of free permits in the following proposition.

PROPOSITION 1

a) *Firm 1's output (abatement) is less (more) sensitive than firm 2's to its own permit allocation.*
b) *When firm* 1 *receives more (less) free permits than firm* 2, *it produces more (less) output and consequently its gross emissions are higher (lower) than firm 2's. It abates less (more) than firm 2 and it becomes a net seller (buyer) of permits.*
c) *The firm that receives more free permits makes a higher profit*

Statement c) in Proposition 1 implies that, if the dominant firm is endowed with less permits than the follower, its advantage due to the market power position is not enough to compensate the disadvantage associated to the initial allocation in terms of profit, although it will serve to strengthen its advantage (when endowed with more permits) or weaken its disadvantage (when endowed with less permits).

In order to have an insight about the relevance that the permit market power has on the output market, it is illustrative to compare this case with the one in which there is not a dominant firm in the permit market (so the permit market is competitive) and the firms still compete a la Cournot. In such a case, total output is given by

$$X = \frac{2+S}{4}$$

and it is straightforward to check that, whenever $\alpha < 0.5 (> 0.5)$, total output is larger (smaller) if there is a dominant firm than if there is not. When $\alpha = 0.5$ output is the same in both cases because the dominant firm cannot exert its market power in the permit market and the result is the same as if the permit market were competitive. Moreover, as expected, the price of permits with a dominant firm is higher than without a dominant firm if the dominant firm receives more permits than the follower ($\alpha > 0.5$).[6]

5. Stackelberg Model with the same Leader in Both Markets

In this section we assume that firm 1 is a dominant firm, not only in the emission permits market, but also in the product market. To some extent, our approach is similar to the one followed by Hintermann (2011) but with a couple of significant differences. First, Hintermann considers a dominant firm and a competitive fringe in the output while we are analyzing a duopoly. Thus, in the output market, the follower acts strategically according to its reaction curve, while in the Hintermann's approach the

[6] Without a dominant firm the price of permits would be given by $\frac{2-3S}{8}$.

fringe firms are price takers. A second difference is that, by using specific demand and cost functions, we are able to obtain a closed solution.

Now, in the output stage, firm 1 sets its output acting as a Stackelberg leader and, afterwards, firm 2 sets its output. The permit stage has the same structure as in the previous version.

We solve the output model by standard methods: the leader chooses its output taking into account the reaction function of firm 2, which is given by (9). Solving the leader's problem and substituting into the follower's reaction function we obtain the equilibrium output:

$$x_1 = \frac{167 + \left(159\alpha + 4\right)S}{538}, \tag{21}$$

$$x_2 = \frac{61 + \left(82 - 103\alpha\right)S}{269}, \tag{22}$$

$$X = \frac{289 + \left(168 - 47\alpha\right)S}{538}. \tag{23}$$

Comparing the expressions for the individual outputs, we conclude that firm 1 tends to produce output more than firm 2 as it is the case in a standard Stackelberg model. This result is reinforced by the fact that firm 1 enjoys a double leadership position in the permit and the output market. From equations (21) and (22) it is straightforward to show that $x_1 > x_2$ holds when both firms receive the same amount of permits ($\alpha = 0.5$), but it can be shown that firm 1 can still produce more than firm 2 even with very low values of α, i.e., with very asymmetric distributions allocating many more permits to firm 1 than to firm 2. The following proposition states that, if the total amount of permits is small enough, firm 1 will always produce more than firm 2 for any allocation of permits (including $\alpha = 0$).

PROPOSITION 2

If firm 1 acts as a leader both in the output and the permit market, there exists a threshold, $\bar{S}^{S1} > 0$*, such that,*

a) If $S > \bar{S}^{S1}$*, there exists a threshold* $\bar{\alpha}^{S1} < 0.5$*, such that firm 1 produces more output than firm 2 if and only if* $\alpha > \bar{\alpha}^{S1}$*.*

b) if $S \leq \bar{S}^{S1}$*, firm 1 produces more output than firm 2 for any allocation of permits* $\alpha \in (0,1]$.

The only way to counterbalance the strong leadership position of firm 1 is to allocate more permits to firm 2. Nevertheless, if the total number of permits is too small, they will not generate a large enough counterbalancing effect so as to compensate for the market power of firm 1. Proposition 3 shows a related result for firms' profits:

PROPOSITION 3

Consider that firm 1 is a Stackelberg leader in the output market and the permit market. Then, there exist two thresholds, $\tilde{S}_L^{S1} > 0$ *and* $\tilde{S}_H^{S1} > 0$ *with* $\tilde{S}_L^{S1} < \tilde{S}_H^{S1}$ *such that*

a) *If* $\tilde{S}_L^{S1} < S < \tilde{S}_H^{S1}$*, there exists a threshold* $\tilde{\alpha}^{S1} \in (0, 0.5)$ *such that the profit of firm 1 is larger than that of firm 2 iff* $\alpha > \hat{\alpha}^{S1}$*.*

b) *If* $S \leq \tilde{S}_L^{S1}$ *or* $S > \tilde{S}_H^{S1}$*, then the profit of firm 1 is larger than that of firm 2 for any value of* $\alpha \in [0,1]$*.*

Proposition 3 states that, apart from producing more, firm 1 also tends to make a larger profit than firm 2, as could be expected from its leadership position. This is always the case for a symmetric distribution of permits ($\alpha = 0.5$), but it could also be true for very asymmetric permit allocations, even for $\alpha = 0$ if the total number of permits is very small, where a similar intuition of that offered for output holds for profit.

The novelty here is that it also holds when the number of permits is very large. The intuition is that, when the number of permits is very large, the permit market does not represent a very strong constraint for firms and, thus, the environmental policy does not have a strong impact on the output market. In that case, the leadership position in the output market is enough to render the dominant firm a higher profit. For intermediate amounts of permits, not very small and not very large, it is possible to find permit allocations (with $\alpha < 0.5$) that would make the follower better off.

As a consequence of Propositions 2 and 3, the $S - \alpha$ space is divided in four regions, as illustrated in Figure 1.

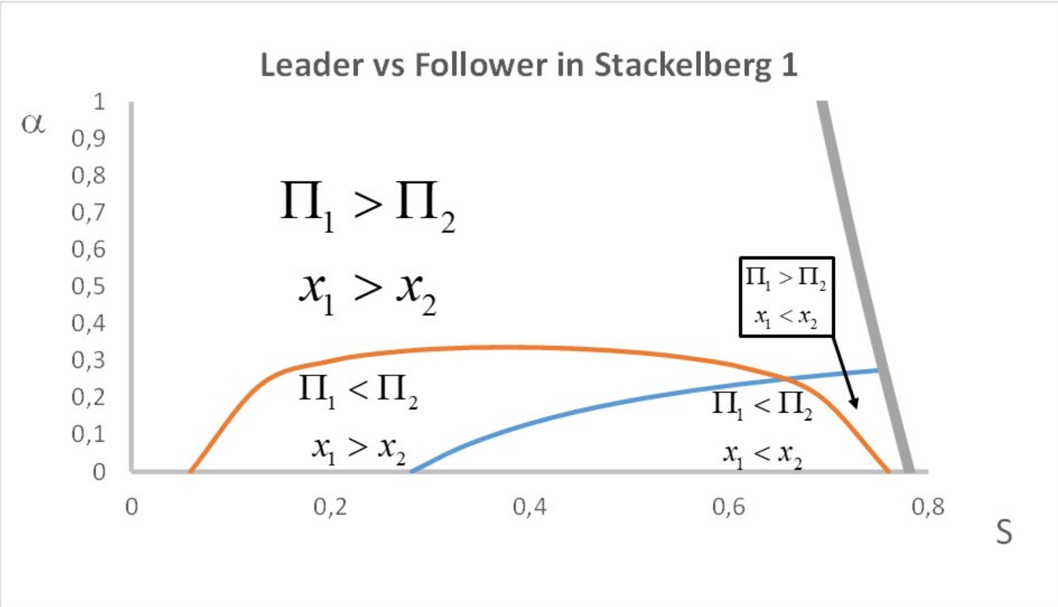

FIGURE 1. Output and profit comparison (leader vs. follower) in model Stackelberg 1 (Propositions 2 and 3). The grey straight line delimits the acceptable combinations of α and *S* (in the sense that *S*<*X*).

Direct comparison with the Cournot case reveals that total output in equilibrium (and hence, abatement and the permit price) will always be higher with a dominant firm than under the Cournot structure, which is a standard microeconomic result. Proposition 4 states that this result holds when the output leader is also the leader in the output market and is robust to the specific permit allocation.

PROPOSITION 4

If firm 1 acts as a leader both in the output and the permit market, total output in equilibrium is higher than in the Cournot model for any allocation of permits, $\alpha \in [0,1]$.

It is also of interest to ask what are the implications of the double leadership position (output and permits) as compared to the more traditional Stackelberg model with just an output leader (without market power in the permit market). It can be shown that, if the permits are allocated evenly ($\alpha = 0.5$), total equilibrium output is lower if there is a firm that dominates both markets as compared to the situation only with an output leader and a competitive permit market. The reason is that the possibility to dominate the permit market increases the capacity of firm 1 to manipulate the market to own profit. Moreover, as we show in the next proposition, if the total number of permits is small enough, the result holds for any permit allocation:

PROPOSITION 5

Consider that firm 1 is a Stackelberg leader in the output market and consider two alternative scenarios: one in which firm 1 also dominates the permit market and another one in which the permit market is competitive. Then, there exists a threshold, $\hat{S}^{S1} > 0$ *such that*

a) If $S > \hat{S}^{S1}$*, then there exists a threshold* $\hat{\alpha}^{S1} \in (0, 0.5)$ *such that total output is smaller with than without market power in the permit market iff* $\alpha > \hat{\alpha}^{S1}$.

b) If $S \leq \hat{S}^{S1}$*, total output is smaller with than without market power in the permit market, for any value of* $\alpha \in [0,1]$

c) If $\hat{\alpha}^{S1} \geq 0.5$*, total output is smaller with market power in the permit market for any number of permits.*

6. A Stackelberg Model with a Different Leader in Each Market

In this section we analyze another situation where one firm dominates the product market and the other one dominates the permit market. The rest of the elements of the model are the same as in the previous section. The main purpose of this version is to ascertain what leadership (output of permits) is preferable for firms and how the interaction between both leaderships impacts on the output market.

For consistency with the previous sections, we denote as "Firm 2" the one that acts as a leader in the output market (and as a price taker in the permit market) while "Firm 1" is still the dominant firm in the permit market and act as a follower in the output market.

Thus, the game has four stages that develop as follows:

1. Firm 2 sets its output acting as a Stackelberg leader
2. Firm 1 sets its output as a follower
3. Firm 1 decides its abatement level (and thus its demand for permits) and the price of permits anticipating the reaction of Firm 2.
4. Firm 2 decides its level of abatement and demand for permits acting as a price taker.

The model is solved by backward induction. Stages 3 and 4 have already been solved in Section 3. By standard methods we solve the output stage and get the following equilibrium values:

$$x_1 = \frac{3 + (5\alpha - 1)S}{14} \tag{24}$$

$$x_2 = \frac{9 + (11 - 13\alpha)S}{28} \tag{25}$$

$$X = \frac{15 + (9 - 3\alpha)S}{28} \tag{26}$$

Direct inspection of the equilibrium values shows that, if the permits are allocated evenly ($\alpha = 0.5$), firm 2 produces more output. Moreover, as it is shown in the following proposition if the total amount of permits is not very large, firm 2 will always produce more than firm 1:

PROPOSITION 6

If firm 1 is a leader in the permit market and firm 2 is a leader in the output market, and $S \leq \frac{3}{10}$, in equilibrium firm 2 produces more output than firm 1 for any allocation of permits, $\alpha \in [0,1]$. If $S > \frac{3}{10}$ there exists a threshold, $\bar{\alpha}^{S2} > 0.5$, such that firm 2 produces more output than firm 1if and only if $\alpha < \bar{\alpha}^{S2}$.

Proposition 6 reveals that, under a symmetric allocation of permits, the leadership position of firm 1 in the permit market cannot reverse, *per se*, the usual result that the Stackelberg output leader produces more output than the follower. In this respect, the leadership position in the output market seems more relevant than the leadership position in the permit market. The only way that the permit leader can produce more than the output leader is by means of an asymmetric distribution of permits (a high value of α) and only if the total number of permits is large enough. Proposition 7 shows a related result about profit.

PROPOSITION 7

If firm 1 is a leader in the permit market and firm 2 is a leader in the output market there exist a threshold, $\tilde{S}_L^{S2} > 0$ *such that*

a) If $\tilde{S}_L^{S2} < S < 1$*, there exists a threshold* $\tilde{\alpha}^{S2} \in (0, 0.6)$ *such that the profit of firm 2 is larger than that of firm 1 iff* $\alpha < \hat{\alpha}^{S2}$.

b) If $S \leq \tilde{S}_L^{S2}$*, then the profit of firm 2 is larger than that of firm 1 for any value of* $\alpha \in [0,1]$.

Summing up, under equal allocation of emission permits, firm 2's output and profit are larger than those of firm 1 due to its dominant position in the output market. The leadership position of firm 1 in the permits is not enough to overcome the output leadership unless firm 2 is endowed it a larger enough amount of permits than firm 2. As shown in Figure 2 there is a range of permits allocation where firm 1 can use its advantage in the permit market to make more profits than firm 2 although it still produces less output due to firm 1's advantage in the product market.

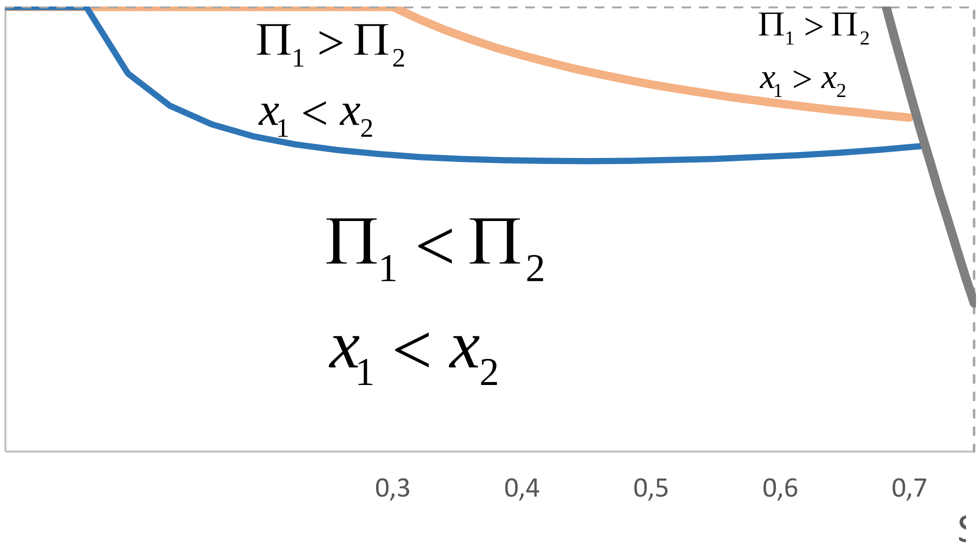

FIGURE 2. Output and profit comparison (firm 1 vs firm 2) in model Stackelberg 2 (Propositions 6 and 7). The grey line delimits the acceptable combinations of α and S (in the sense that S<X).

7. Comparison across models

In this section, we care about the overall results of all three models in terms or output and welfare. The next proposition compares the three models in terms of impact in the output market.

PROPOSITION 8

a) *If firm 1 is a leader in the permit market and firm 2 is a leader in the output market total output in equilibrium is higher than in the Cournot model for any allocation of permits* $\alpha \in [0,1]$.

b) *There exist a threshold* $\tilde{S}_L^{S12} > 0$ *such that if* $S \leq \tilde{S}_L^{S12}$, *then the total output in equilibrium is higher in model Stackelberg 1 than in model Stackelberg 2 for any value of* $\alpha \in [0,1]$.

c) If $\tilde{S}_L^{S12} < S < 1$, *there exist a threshold* $\tilde{\alpha}^{S2} \in (0, 0.5)$ *such that the total output in equilibrium is higher in model Stackelberg 1 than in model Stackelberg 2 iff* $\alpha > \hat{\alpha}^{S12}$

Statement a) in Proposition 8 restates the classical result that output in a Stackelberg model is higher than in a Cournot model and this is true for any distribution of the permits. This is a feature that model Stackelberg shares with model Stackelberg 1 (see Proposition 4 above), which implies that this feature of the model is not affected by the way in which the leaderships (in output and permits) are distributed between the firms.

Statements b) and c) are illustrated in Figure 3. The main conclusion is that output tends to be larger in model Stackelberg 1 than in Stackelberg 2. To get the intuition behind this result it is useful to recall the relationship between the classical Cournot and Stackelberg models. In Stackelberg, the leader uses its advantageous position to produce more and make more profit, which ultimately results in a larger total output in the market. In our model Stackelberg 1, this mechanism is reinforced by the double leadership position of a single firm. Nevertheless, in Stackelberg 2, the position of the output leader is somewhat undermined by the permit power of the competing firm. Nevertheless, the output leadership tends to prevail over the permit leadership except if the number of permits is large enough and a large enough share of them are allocated to the permit leader.

Finally, we care about the welfare comparison across scenarios. We measure welfare by adding up consumers' surplus and the profit of both firms. The third component that should be included in total surplus is (minus) the social welfare damage caused by pollution. Nevertheless, since total emissions is equal to S by construction in all three models, we can concentrate of the two first components (consumers' surplus and firms' profit). The resulting welfare is compared numerically. Total welfare in any of the Stackelberg scenarios turns out to be larger than in Cournot for any permit distribution, mainly due to the fact that output is larger in Stackelberg model. The comparison between the two Stackelberg versions are illustrated in Figure 3.

In those cases in which output is higher in one of the models, aggregate welfare also tends to be higher. Nevertheless, there is a range of permit allocations such that welfare is higher in Stackelberg 2 although output is higher in Stackelberg 1. The reason is that, in those cases, the leadership position of firm 1 in the permit market helps to counterbalance the leadership position of firm 2 in the output market in such a way that abatement effort (and thus, abatement cost) is distributed more evenly between firms, which results in lower total cost and, thus, a higher overall profit of the firms (which, in this case, more than compensates the lower consumers' surplus due to lower production).

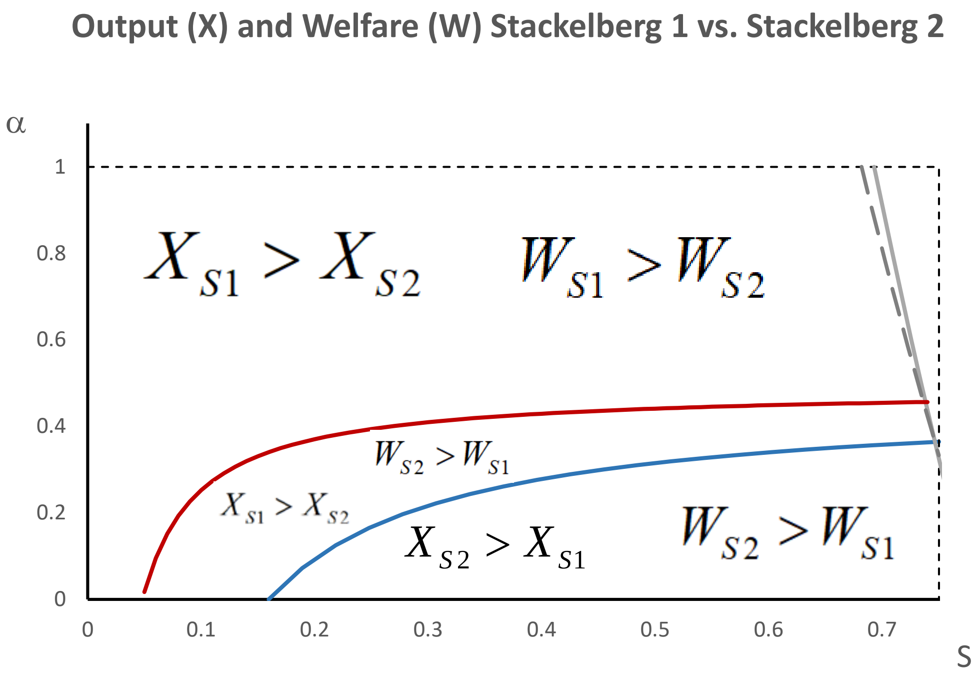

FIGURE 3. Output and welfare comparison between Stackelberg 1 and 2. The grey solid and dashed lines delimit the acceptable combinations of α and S (in the sense that S<X) for Stackelberg 1 and Stackelberg 2 respectively.

8. Concluding Remarks

In this paper we have considered three different oligopolistic structures in the product market under the assumption of imperfect competition in the related permit market. The main focus of our analysis has been to analyze the role of permit distribution in the outcomes of firms in terms of output and profits and also the implications for global welfare.

For the Cournot model, we have shown that if both firms initially receive the same amount of permits, there will not be any permit trade. Both firms will produce the same quantities and make the same profit and therefore firm 1´s dominant position in the emission permits market does not lead to any competitive advantage. If the allocation is not symmetric, the firm who receives more free permits produces a higher quantity and makes higher profits. We also show that, when endowed with more permits, the permit leader tends to increase output less and reduce abatement more than the follower firm in the same situation.

In both Stackelberg models we got the standard result that output is larger than in the Cournot model, which is strengthened by the fact that it holds for any output allocation.

When there is a single leader in both markets, it tends to produce more and make more profit than its rival, although each one of these implications (or both) can be reversed if enough permits are allocated to the following firm.

From the Stackelberg model with two different leaders, one in each market, we conclude that the output leadership tends to be more relevant as the output leader produces more and makes more profits than the dominant firm in the permit market. This is true under a symmetric allocation of permits but as soon as we consider that the permit market leader is receiving more permits, both firms output tends to equalize first and it comes to a point where the Stackelberg leader in the product market produces less and makes less profits than the follower.

Any of the two Stackelberg models results in more output and social welfare than Cournot. The case in which firm 1 leads both markets simultaneously tends to generate more output and more welfare than the case with two different leaders except if enough permits are allocated to the permit leader. In that case, the permit leadership somewhat compensates the output leadership and Stackelberg 2 can result in larger output and welfare.

References

André, F.J. and de Castro, L.M. (2017) "Scarcity climate rents under a carbon price with oligopoly competition" Oxford Economic Papers 69(2): 470-495.

Baumol, WJ. & Oates, W.E. (1971) "The use of standards and prices for protection of the environment" The Swedish Journal of Economics 73 (1): 42-54.

Bernard, A., S. Paltsev, J.M. Reilly, M. Vielle and L. Viguier (2003) "Russia's Role in the Kyoto Protocol", Report No. 98. The MIT Joint Program on the Science and Policy of Global Change.

Chevallier, J. (2008) "Strategic Manipulation on Emissions Trading Banking Program with Fixed Horizon." Economics Bulletin 17(14): 1-9.

De Feo, G., Resende, J., and Sanin, M.E. (2013) Emission permit trading and downstream strategic market interaction, The Manchester School, 81, 780–802.

Fowlie, M. (2010) "Emissions Trading, Electricity Restructuring and Investment in Pollution Abatement", American Economic Review 100: 837-869

Hagem, C. and Maestad, O (2006) "Russian Exports of Emission Permits under the Kyoto Protocol: The Interplay with Non-competitive Fuel Markets." Resource and Energy Economics 28: 54-73.

Hahn, R. (1984) "Market Power and Transferable Property Rights" Quarterly Journal of Economics 99: 753-765.

Hintermann, B. (2011) "Market Power, Permit Allocation and Efficiency in Emission Permit Markets", Environmental and Resource Economics 49: 327-349

Hintermann, B. (2017) "Market Power in Emission Permit Markets: Theory and Evidence from the EU ETS", Environmental and Resource Economics 66: 89-112

Kolstad, J. and Wolak, F. (2003) “Using Environmental Emissions Permit Prices to Raise Electricity Prices: Evidence from the California Electricity Market” University of California Energy Institute, CSEM WP 113.

Misiolek, W. and Elder, H. (1989) “Exclusionary manipulation of markets for pollution rights”, Journal of Environmental Economics and Management 16: 156–66.

Montero, J.P. (2009) “Market Power in Pollution Permit Markets”, The Energy Journal 30 (special issue 2): 115-142.

Montgomery, W. D. (1972) “Markets in Licenses and Efficient Pollution Control Programs”, Journal of Economic Theory 5: 395–418.

Sartzetakis, E.S. (1997) “Tradeable emission permits regulations in the presence of imperfectly competitive product markets: welfare implications”, Environmental and Resource Economics 9, 65-81.

Tanaka, M. & Chen, Y (2012) “Market power in emissions trading: Strategically manipulating permit price through fringe firms” Applied Energy 96: 203-211.

Tietenberg, T.H. (2006) "Emissions Trading. Principles and Practice", second edition. Resources for the Future, Washington D.C. USA.

APPENDIX

Leader's solution in the permit market

The leader's objective function in the permit market can be written as

$$\frac{q_1^2}{2} + py_1 = \frac{q_1^2}{2} + q_2\left(x_1 - \alpha S - q_1\right)$$

where we have used (2) and firm 1's permit constraint, $y_1 = x_1 - \alpha S - q_1$. Combining the permit constraint for both firms and the market-clearing condition, $y_2 = -y_1$, we get $q_2 = x_2 + x_1 - q_1 - S$ and then the leader's problem can be written as

$$\min_{\{q_1\}} \frac{q_1^2}{2} + \left(x_1 + x_2 - q_1 - S\right)\left(x_1 - \alpha S - q_1\right)$$

and solving the first order condition of this problem we get (5).

Proof of Proposition 1

Statement a) has been proved in the main text (see equations (17) and (18)). Statement b) immediately follows from (13), (16) and (7). To prove statement c) note that the difference in profits can be written as

$$\begin{aligned}\Pi_1 - \Pi_2 &= P\left(x_1 - x_2\right) - \frac{1}{2}\left(q_1^2 - q_2^2\right) - 2py_1 \\ &= P\left(x_1 - x_2\right) - \frac{1}{2}\left(q_1 + q_2\right)\left(q_1 - q_2\right) - 2py_1\end{aligned} \qquad \text{(A1)}$$

where P denotes the output price and we have used the permit market clearing condition $y_1 = -y_2$. From equations (5) and (6) we get $q_1 + q_2 = x_1 + x_2 - S$ and from (7) we know $y_1 = q_1 - q_2$. Using these results, (A1) can be written as

$$\Pi_1 - \Pi_2 = P\left(x_1 - x_2\right) - y_1\left(\frac{1}{2}\left(q_1 + q_2\right) + 2p\right) \qquad \text{(A2)}$$

where the term in brackets that multiplies y_1 in (A2) is clearly positive. Consider $S_1 > S_2$. Statement b) in this proposition implies that the first term of the right-hand side in (A1) is positive and, since $y_1 < 0$, the second part is also positive (as it is affected by a minus sign). Therefore, $\Pi_1 > \Pi_2$. The opposite case can be proved in a similar way.

Proof of Proposition 2

Combining (21) and (22), the difference in output is given by

$$x_1 - x_2 = \frac{45}{538} + \frac{(365\alpha - 160)S}{538} > 0 \Leftrightarrow 45 + (365\alpha - 160)S > 0 \qquad \text{(A3)}$$

so, $x_1 > x_2$ except if $(365\alpha - 160)S$ is negative enough, which requires α to be small enough and S large enough. Define $\overline{S}^{S1} := \frac{9}{32}$. It is easy to show that, for any $S < \overline{S}^{S1}$, (A3) is positive even for $\alpha = 0$, which is the most adverse case.

Consider now $S > \overline{S}^{S1}$. Solving (A3) for α we conclude

$$x_1 > x_2 \Leftrightarrow \alpha > \frac{160S - 45}{365S} := \overline{\alpha}^{S1}.$$

Proof of Proposition 3

Define the difference in profit of both firms as $\Delta\Pi := \Pi_1 - \Pi_2$ Using the equilibrium values of output and abatement in the profit functions of both firms, after some algebra we arrive at the following expression:

$$\Delta\Pi = \frac{9855 + (379374\alpha - 179832)S + (113231\alpha^2 - 502432\alpha + 225372)S^2}{578888} \qquad \text{(A4)}$$

The sign of (A4) is determined by the numerator, which is a convex function of α. Moreover, it is increasing in α for any acceptable values of α and S. This implies that the minimum value of $\Delta\Pi$ is attained when $\alpha = 0$. If we set $\alpha = 0$ in (A4) we get

$$\Delta\Pi\left(\alpha=0\right)=\frac{9855-179832S+225372S^{2}}{578888},$$

which is a U-shaped function of S. By equating the numerator to zero we get a second order equation whose roots are given by $\tilde{S}_L^{s1}:=0.05921$ and $\tilde{S}_H^{s1}:=0.73873$. Below $\tilde{S}_L^{s1}$ and above $\tilde{S}_H^{s1}$ we have $\Delta\Pi>0$ even for $\alpha=0$, and thus for any acceptable value of α, as $\Delta\Pi$ is increasing in α. Consider now $\tilde{S}_L^{s1}<S<\tilde{S}_H^{s1}$.

Equating the numerator of (A4) to zero we get a second order equation with a positive and a negative root. Since α cannot be negative, we take the positive one, which is given by

$$\tilde{\alpha}^{S1}=\frac{251216S-189687+538\sqrt{129871S^{2}-258918S+120456}}{113231} \tag{A5}$$

Since $\Delta\Pi$ is increasing in α, we conclude that $\Delta\Pi>0$ holds whenever $\tilde{S}_L^{s1}<S<\tilde{S}_H^{s1}$ and $\alpha>\tilde{\alpha}^{S1}$. For a symmetric allocation ($\alpha = 0.5$) the value of (A4) is

$$\Pi=\frac{1}{578888}\left(9855+\frac{9855}{4}S^{2}+9855S\right) \tag{A6}$$

which is strictly positive for any value of S. Therefore, $\tilde{\alpha}^{S1}$ must be lower than 0.5.

Proof of Proposition 4

Direct comparison of (19) and (23) shows that total output under the Stackelberg-1 model is larger than that under the Cournot model if and only if $140+100S-60\alpha S>0$, which is true for any acceptable values of S and α.

Proof of Proposition 5

When the permit market is competitive, instead of equations (2)-(5) we get

$$
\begin{aligned}
&p = q_1 = q_2 \\
&y_1 = x_1 - S\alpha - p \\
&y_2 = x_2 - (1-\alpha)S - p
\end{aligned}
\tag{A7}
$$

The market clearing condition implies that the total number of permits equals total net emissions, i.e., $S = x_1 - q_1 + x_2 - q_2 = x_1 + x_2 - 2p$, from which get the equilibrium permit price

$$
p = q_1 = q_2 = \frac{x_1 + x_2 - S}{2} \tag{A8}
$$

By solving the follower's problem in the output market and plugging its reaction function in the leader's problem we get the equilibrium output values:

$$
\begin{aligned}
x_1 &= \frac{61 + S(63\alpha - 1)}{189} \\
x_2 &= \frac{41 + S(52 - 63\alpha)}{189} \\
X &= \frac{34 + 17S}{63}
\end{aligned}
\tag{A9}
$$

Now, using (23) and (A9), we can compare total output with and without competition in the permit market

$$
\Delta X := X_{MPPM} - X_{CPM} = \frac{(1438 - 2961\alpha)S - 85}{33894} \tag{A10}
$$

where *MPPM* and *CPM* refer to "market power in the permit market" and "competitive permit market respectively. (A10) shows that $X_{MPPM} < X_{CPM}$ except if α is small enough and S is large enough. Specifically, $X_{MPPM} < X_{CPM}$ holds for any $\alpha > {}^{1438}\!/_{2961} \simeq 0,49$, whatever the number of permits. Take $\alpha = 0$, which is the most favorable case for $X_{MPPM} > X_{CPM}$. Solving for S, we conclude that, whenever $S < \hat{S}^{S1} := \frac{85}{1438}$, $X_{MPPM} < X_{CPM}$ even for $\alpha = 0$, and thus for any acceptable value of α. If, on the contrary, $S > \hat{S}^{S1}$, it is immediate to show that $X_{MPPM} < X_{CPM}$ when

$$\alpha < \hat{\alpha}^{S1} := \frac{1438S - 85}{2961S} < 0.5$$

Proof of Proposition 6

The difference in output can be obtained from equations (24) and (25) as given by

$$x_2 - x_1 = \frac{3 + S(13 - 23\alpha)}{28}, \tag{A11}$$

which is decreasing in α. Taking the most favorable value of α for the difference to be negative, $\alpha = 1$, and solving for S we conclude that, whenever $S \leq \frac{3}{10}$, $x_2 \geq x_1$ even for $\alpha = 1$, and thus for any acceptable value of α. If $S > \frac{3}{10}$ we conclude

$$x_2 - x_1 \geq 0 \Leftrightarrow \alpha \leq \frac{13S + 3}{23S} > 0.5.$$

Proof of Proposition 7

The difference in profits can written as

$$\begin{aligned} \Delta\Pi = \Pi_1 - \Pi_2 &= (1 - X)(x_1 - x_2) - \frac{1}{2}(q_1^2 - q_2^2) - 2py_1 \\ = \Pi_1 - \Pi_2 &= (1 - X)(x_1 - x_2) - \frac{1}{2}(q_1 + q_2)(q_1 - q_2) - 2py_1 \end{aligned} \tag{A12}$$

or, using (7),

$$\Delta\Pi = (1 - X)(x_1 - x_2) - (q_1 - q_2)\left[\frac{1}{2}(q_1 + q_2) + 2p\right]. \tag{A13}$$

and using equations (2), (5), (6), (7), (24) and (25) and rearranging, we get

$$\Delta\Pi = \frac{(281\alpha^2 - 1294\alpha + 569)S^2 + (1110\alpha - 586)S - 31}{1568}, \tag{A14}$$

which is a convex function of α. Solving $\Delta\Pi = 0$ for α we conclude that there is only a positive (and, thus, acceptable) root, given by

$$\tilde{\alpha}^{S2} = \frac{647S - 555 + 28\sqrt{330S^2 - 706S + 404}}{281S} \tag{A15}$$

and, by noting $\Delta\Pi\left(\alpha = 0\right) < 0$ it is immediate to check that, below (above) the threshold given by (A15), we have $\Pi_1 < \Pi_2$ ($\Pi_2 < \Pi_1$). For a symmetric allocation $\alpha = 0.5$ we have

$$\Delta\Pi = \frac{-31}{1568} - 0.00494S^2 - 0.0197S \tag{A17}$$

which is negative for any acceptable value of S and thus, (A15) must be larger than 0.5. Direct inspection shows that, for any value of S below $\tilde{S}_L^{S2} \simeq 0.06$, (A15) is larger than 1, which means that, for any acceptable value of α we have $\Pi_1 < \Pi_2$.

Proof of Proposition 8

Using (16) and (26), the difference in output between Cournot and Stackelberg 2 is given by

$$X_C\text{-}X_{S2} = \frac{\left(\alpha - 1\right)S - 1}{28} \ ,$$

which is negative for any acceptable values of α and S. This proofs statement a).

Regarding statements b) and c), use (23) and (26) to obtain the difference in profits between both models:

$$X_{S1}\text{-}X_{S2} = \frac{\left(149\alpha - 69\right)S + 11}{7532},$$

which is positive if and only if $\alpha > \tilde{\alpha}^{S12} := \frac{69S - 11}{149S}$. But this threshold turns out to be negative whenever $S < \tilde{S}_L^{S12} := \frac{11}{69}$. In such a case $X_{S1} > X_{S2}$ holds for any acceptable value of α.