Emission Taxes, Feed-in Subsidies and the Investment in a Clean Technology by a Polluting Monopoly*

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Abstract

The paper studies the use of emission taxes and feed-in subsidies for the regulation of a monopoly that can produce the same good with a technology that employs a polluting input and a clean technology. The second-best tax and subsidy are calculated solving a two-stage policy game between the regulator and the monopoly with the regulator acting as the leader of the game. We find that the second-best tax rate is the Pigouvian tax. The tax implements the efficient level of the dirty output but does not affect the total output. On the other hand, the subsidy leads to the monopoly to reduce the dirty output but also to increase the total output. This increase in total output may yield a larger net social welfare when the subsidy is used provided that the marginal cost of clean output is not very high, as a linear-quadratic specification of the model confirms. Finally, it is showed that the combination of an emission tax with a feed-in subsidy induces the firm to choose the efficient outputs, but in this case the first-best tax must be lower than the Pigouvian tax. Thus, the findings of this paper support the idea that feed-in subsidies open the possibility for improving the regulation of a polluting firm with market power.

Keywords: monopoly, polluting inputs, clean technology, production-mix, emission tax, feed-in subsidy

JEL Classification System: D42, H23, L12, Q58

1 Introduction

Environmental regulation for a polluting monopoly is an interesting case of regulation since the market equilibrium can be inefficient because two market failures are operating at the same time but in an opposite direction. On one hand, the firm's *market power* leads to a contraction of output and emissions below their efficient levels. On the other hand, there is a *negative externality* that has the opposite effects. Consequently, there will exist a level of marginal environmental damage for which the market equilibrium is efficient. Then, if marginal damages are below this threshold, a Pigouvian tax will lead to a reduction in welfare instead of implementing the efficient outcome as was shown by Buchanan (1969). In fact, when the only way to adjust emissions is changing production, the first-best emission tax must be lower than the marginal damage and even it could be negative playing implicitly as a subsidy on production if the marginal damage is low enough. Barnett (1980) analyzes the emission taxation of a monopolist that can operate with an abatement technology that allows abate emissions without reducing production. In this setting he also obtains that the tax that maximize net social welfare must be lower than the marginal damage of emissions, but now the tax by itself cannot implement the efficient outcome because a second policy instrument is needed to adjust the level of abatement investment. Interestingly, it is easy to show using Barnett's (1980) model that when the firm can operate with an abatement technology the first-best policy consists of a combination of a subsidy on production and a Pigouvian tax on emissions. The tax corrects the distortion caused by the negative externality and a subsidy equal to the difference between the price and the marginal revenue adjusts the distortion created by the power market of the firm. The problem with this policy is that a subsidy for a monopoly that also implies a subsidy on dirty production can be seen from a political perspective as an alternative out of the menu of environmental policy instruments. However, this opposition to use a subsidy on output can disappear if the subsidy is on clean output as occurs with the feed-in subsidies for promoting renewable energy sources (RES) deployment. A policy that has become very popular, mainly in Europe, in the last twenty years.[1]

The aim of this paper is to extend the analysis of the environmental regulation for a polluting monopoly to take into account the possibility of using a feed-in subsidy on clean output. The model we propose is that of a monopoly that operates with a technology that uses a polluting input, but that can also invest in a clean technology to produce the same good. A clear example is the production of electricity. Usually, electricity firms work with different technologies that use different inputs yielding an electricity-mix that depends of the way the firm combines the different technologies to produce the total output.

In the first part of the paper, we calculate the monopoly equilibrium and characterize the efficient outcome. In both cases, the firm will invest in the clean technology provided that the polluting input price is not too low. However, the threshold price for the polluting input that triggers the investment in the clean technology is lower for the efficient outcome than for the market equilibrium. Thus, there exists an interval for the input price for which the firm should invest in the clean technology, but it does not find it profitable. When the firm finds profitable to invest in the clean technology, the monopolist's clean production is lower than the efficient level whereas for the dirty production occurs the contrary. As expected, the monopoly total production is lower than the efficient level yielding a larger price. In the second part of the paper, we analyze second-best policies. The focus is on the comparison of feed-in subsidies and taxes. The target is to rank the two policy instruments according to the level of welfare they allow to achieve. First, we characterize the second-best emission tax. Secondly, we analyze the effect that a feed-in subsidy on the investment in a clean technology. With this aim, we solve a two-stage

[1] According to a recent report of the Council of European Energy Regulators (2018) for the 2016-2017 period, 20 out of 28 Member countries of the European Union (EU) were applying this type of subsidies including in this list France, Germany, Italy and the United Kingdom, the main economies of the EU. For the review period of 2016-2017, four types of instruments were mainly in place in Europe, namely: feed-in tariffs (FITs), feed-in premiums (FIPs), green certificates (GCs), and investment grants. In this paper, we focus on the use of FIPs. An interesting paper evaluating policies to deploy RES comparing the experience in the Unites States and the EU is Schmalensee (2012) and a paper assessing the Spanish feed-in tariff at the beginning of the century is del Río and Gual (2007).

policy game between the regulator and the monopolist. In the first stage, the regulator chooses the level of the policy instrument to maximize net social welfare. In the second stage, the firm decides on the use of the clean and dirty technologies. Thus, the optimal policy is given by the Stackelberg equilibrium of the policy game.

Our analysis shows that the Pigouvian tax is the optimal tax in a second-best setting. The tax corrects the distortion caused by the negative externality but have no effect on total output because the long-run marginal cost of clean output is constant. The tax implements the efficient level of dirty output improving the production-mix in favour of the clean technology. On the other hand, the feed-in subsidy increases total output and reduces dirty output and it could implement the efficient outcome, but only under special circumstances. In general, the clean and dirty outputs could be larger or lower than the efficient levels. This leaves the comparison between the emission tax and the feed-in subsidy undetermined, except for the total output. However, if the dirty output is higher when a tax is applied, it is easy to check that with low enough marginal cost for the clean output, the subsidy would yield a higher net social welfare than that obtained with the tax. To advance in the analysis, we give more structure to the model and investigate whether this hypothesis is true for the linear-quadratic model. In this setting, we find that if the clean technology is efficient enough to yield a low marginal cost, the feed-in subsidy leads to a higher clean output and a lower dirty output than those obtained applying a tax. Moreover, total output is larger when a feed-in subsidy is used. Finally, we find out that net welfare is greater for a feed-in subsidy. With an emission tax, the increase in welfare respect to the monopoly equilibrium comes only from the reduction in damages. However, when a subsidy is used we have to add to the decrease in damages an increase in the consumers' surplus along with a rise in the profit because the firm sells a larger output. These positive variations are larger than the increase in costs caused by the reduction in the dirty output yielding a higher net welfare when the environmental policy consists of applying a feed-in subsidy on clean output. The paper ends showing that a combination of an emission tax and a feed-in subsidy implements the efficient outcome. The optimal subsidy is equal to the difference between the price and the marginal revenue, but the optimal tax must be lower than the marginal damage. The Pigouvian tax works with a subsidy on total output as we pointed out above, however if the subsidy only applies on the clean output the tax has to correct not only the negative externality caused by the dirty output, but also the distortion caused by the market power on the dirty output. Then, the tax must be lower than the environmental marginal damage, although now the combination of the tax and the feed-in subsidy leads to the regulated market equilibrium to implement the efficient outcome.

1.1 Literature Review

As we have just commented, the first characterization of the second-best emission tax under monopoly when the firm operates with an abatement technology is due to Barnett (1980). He shows that the optimal tax is lower than the marginal environmental damage. However, he points out that when the tax has no effect on output, the second-best tax is the Pigouvian tax. Our model presents an example of this special case. When the clean technology presents constant returns, the total output does not depend on the tax and the optimal tax is equal to the marginal damage. Barnett's (1980) analysis of the environmental regulation of a polluting monopoly has been extended in different directions among others by Besanko (1987), Katsoulacos and Xepapadeas (1995), Innes and Bial (2002), Montero (2002), Farzin (2003), Petrakis and Xepapadeas (2003), Puller (2006), Poyago-Theotoky (2007, 2010), Canton et al. (2008) and more recently by Moner-Colonques and Rubio (2016) and Martín-Herrán and Rubio (2018). In all these papers, following Barnett's (1980) approach, emissions depend positively on output and negatively on a variable that can stand for both the resources devoted to abatement activities or a coefficient emissions/production that can be reduced at an increasing cost for the firm.[2] The problem with this specification of the emission function is that is not suitable for analyzing an environmental policy based on feed-in subsidies because is not possible to discriminate between clean and dirty outputs. In fact, this literature focuses on the

[2]Montero (2002), Petrakis and Xepapadeas (2003), Poyago-Theotoky (2007), Canton et al. (2008), Moner-Colonques and Rubio (2016) and Martín-Herrán and Rubio (2018) assume an end-of pipe abatement technology. For this kind of technologies, the net emissions are equal to gross emissions that are proportional to the output minus abatement.

use of taxes, different types of standards or tradable permits, but no paper addresses the issue of subsidies on output as an instrument of the environmental policy. In this list, we could also include the papers by Coria (2009), Gil-Moltó and Varvarigos (2013) and Krass et al. (2013) that study the effects of emission taxation on the adoption of a green technology.

More recently, different scholars have analyzed the environmental regulation in a context of imperfect competition considering that the same good can be produced with more than one technology and the regulator applies a feed-in subsidy to promote the use of the clean technology. The list includes the papers by Tamás et al. (2010), Reichenbach and Requate (2012), Sun and Nie (2015), and Fehr and Ropenus (2017).[3] Tamás et al. (2010) study the use of feed-in subsidies and tradable green certificates to achieve a given quota of renewables in a oligopolistic market with clean and dirty firms.[4] Reichenbach and Requate (2012) assume that the dirty firms form an oligopoly whereas the clean firms constitute a competitive fringe. Furthermore they consider an upstream competitive industry producing renewable energy equipment engaged in learning by doing. They show that a first-best policy requires two instruments, a tax on dirty output and feed-in subsidy for renewable energy equipment producers. The tax, that is lower than the environmental damages, corrects for both the externality of pollution and the output contraction due to oligopoly power. The subsidy corrects for insufficient public learning. They also recognize that if emissions are not proportional to output, but firms have a separate clean technology, a Pigouvian tax will only correct for the pollution and a separate subsidy on dirty output would be necessary to correct for the distortion caused by the firms' market power. Our paper shows that with a clean technology, the efficient

[3] We could also mention the papers published by Requate (2015) and Antoniou and Strauz (2017), although they assume that the polluting firms sell their output in a competitive market.

[4] In this paper and also in Reichenbach and Requate (2012) it is assumed that there are two technologies, but that the firms use only one resulting in a market with clean and dirty firms. Baumann and Friehe (2017) also analyze an oligopoly with clean and dirty firms, but they examine the consequences of an increase in the expected fine for non-compliance with an environmental design standard when the number of firms is endogenous.

outcome can be also implemented without using a subsidy on dirty output. The first-best policy would consist of a feed-in subsidy on the clean output and an emission tax, but as the subsidy only applies to a part of total production, the tax should be lower than the environmental damage to complete the corrective effect of the subsidy on the distortion created by the firms' market power. Thus, the use of subsidies on dirty output can be avoid using instead feed-in subsidies on clean output, but then the Pigouvian tax is not the rule for a polluting firm with market power.

Sun and Nie (2015) work with a model very similar to the one used in this paper, but they focus in the numerical comparison of feed-in subsidies versus renewable portfolio standards assuming that the policy objective of the regulator is to achieve an exogenously given market share of the clean output. Our focus is on the comparison of feed-in subsidies versus taxes and on the characterization of the second-best and first-best policies. Finally, we could mention the paper by Fehr and Ropenus (2017) where they compare the market equilibrium with feed-in subsidies and with green certificates of an industry in which a dominant firm producing both dirty and clean output is facing a competitive fringe of clean firms. They demonstrate that markets for green certificate allow the dominant firm to squeeze the margins of its competitors, but that this does not occur when a system of feed-in subsidies is used.

The paper is organized as follows. In the next section, Section 2, we present the model and characterize the market equilibrium and the efficient outcome. In Section 3 we calculate the second-best emission tax and the second-best feed-in subsidy and compare the outcomes of the regulated market equilibrium. The comparative analysis is completed in Section 4 using a linear-quadratic model. The first-best policy is characterized in Section 5. Section 6 closes the paper with the conclusions and the presentation of different issues for future research.

2 The Model

We consider a monopoly that faces a market demand represented by the decreasing inverse demand function $p = D(q)$, where q is the firm's output and $D' < 0$. Moreover, we assume that the marginal revenue is decreasing with the output what requires that $D''q + 2D' < 0$. The firm can produce the output using a *dirty* technology that employs a polluting factor given by $q_d = f(e)$, where e stands for the polluting input with $f(0) = 0$, $f' > 0$, $\lim_{e \to 0} f' = +\infty$ and $f'' < 0$. After an appropriate choice of measurement units we can say that each unit of input generates one unit of pollution.[5] According to this technology, the production cost of the dirty output is $C_d(q_d) = p_e e(q_d)$, where p_e is the input market price for the polluting input. As $e' = 1/f' > 0$ and $e'' = -1/f'f'' > 0$, the marginal cost of the dirty output is zero when output is zero and increases with the quantity.[6] Pollution generates environmental damages given by the function $ED(e)$, $ED' > 0$, $ED'' \geq 0$. Alternatively, the firm can also produce the same good investing in a *clean* technology that operates with constant returns $q_c = ak$, and costs p_k. Thus, the long-run production cost of the clean technology is $C_c(q_c) = \beta q_c$, where β is equal to p_k/a. Total output of the firm is given by $q = q_c + q_d$ and we assume that $\beta < D(0)$.

The market regulation is analyzed as a *policy game* where the firm chooses the quantities to maximize its profits and the regulator selects the policy with the aim of maximizing net social welfare. Before studying the outcome of this game, we will characterize the monopoly equilibrium and the efficient allocation.

[5] A first paper investigating the effects of an emission tax on the incentives for oligopolists to acquire a clean technology where pollution is connected explicitly with the use a production factor is Damania (1996).

[6] The analysis could be extended to consider other inputs assuming that the technology presents decreasing returns to scale. This specification of the production function could also represent a situation where there exist different polluting technologies operating each one with a constant marginal cost. Then, the increasing marginal cost used in the model could be seen as an approximation of the polluting technologies. An example could be the electricity production for which different polluting inputs as coal, fuel oil and gas can be burned for its production.

2.1 The Monopoly Equilibrium

First, we analyze the *short-run* decision on output. In the short run, the maximization problem faced by the monopolist is the following:

$$\max_{\{q_c,q_d\}} \pi = D(q)q - p_k k - p_e e(q_d),$$

$$s.t.\ q = q_c + q_d, \quad q_c \leq ak.$$

The Kuhn-Tucker conditions of the problem are

$$\frac{\partial \mathcal{L}}{\partial q_c} = D'(q)q + D(q) - \lambda \leq 0,\ q_c \geq 0,\ q_c \frac{\partial \mathcal{L}}{\partial q_c} = 0, \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial q_d} = D'(q)q + D(q) - p_e e'(q_d) \leq 0,\ q_d \geq 0,\ q_d \frac{\partial \mathcal{L}}{\partial q_c} = 0, \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = ak - q_c \geq 0,\ \lambda \geq 0,\ \lambda(ak - q_c) = 0. \tag{3}$$

As the short-run marginal cost (SMC) of the clean technology is zero, if the marginal revenue is zero for a positive quantity, $\hat{q}$, the firm could produce all output only with the clean technology provided that the installed capacity allows to the firm produces a quantity equal to or higher than $\hat{q}$. Then, the multiplier should be zero since it must satisfied that

$$\lambda = D'(ak)ak + D(ak) \leq p_e e'(0) = 0 \text{ and } \lambda \geq 0.$$

Instead suppose that the firm uses both technologies what implies that $q_c = ak$ and

$$D'(ak + q_d)(ak + q_d) + D(ak + q_d) - p_e e'(q_d) = 0, \tag{4}$$

which implicitly defines the function $q_d(k)$. According to Implicit Function Theorem, the slope of this function is

$$\frac{dq_d}{dk} = -\frac{a(D''q + 2D')}{D''q + 2D' - p_e e''} < 0, \tag{5}$$

because we have assumed that the marginal revenue is decreasing. Then, the impact of a change in k on total output will be

$$\frac{dq}{dk} = \frac{dq_c}{dk} + \frac{dq_d}{dk} = a - \frac{a(D''q + 2D')}{D''q + 2D' - p_e e''} = \frac{-ap_e e''}{D''q + 2D' - p_e e''} > 0. \tag{6}$$

Next, we analyze the *long-run* decision on investment in the clean technology. The firm will choose the level of capital that maximizes its profits taking into account the effect that the decision has on output

$$\max_{\{k\}} \pi = D(ak + q_d(k))(ak + q(k)) - p_k k - p_e e(q_d(k)).$$

The Kuhn-Tucker condition of this problem is

$$\frac{\partial \pi}{\partial k} = (D'(q(k))q(k) + D(q(k)))\left(a + \frac{dq_d}{dk}\right) - p_k - p_e e'(q_d(k))\frac{dq_d}{dk} \leq 0,$$

$$k \geq 0,\ k\frac{\partial \pi}{\partial k} = 0,$$

that according to condition (4) results in

$$\frac{\partial \pi}{\partial k} = (D'(q(k))q(k) + D(q(k)))a - p_k \leq 0,\ k \geq 0,\ k\frac{\partial \pi}{\partial k} = 0.$$

Let's suppose that there is a complete substitution of the dirty technology by the clean one. Then,

$$D'(ak)ak + D(ak) = \frac{p_k}{a} = \beta = \lambda > 0,$$

what contradicts our previous result that $\lambda = 0$ when this occurs. Thus, although the SMC of the clean output is zero, the long-run marginal cost (LMC) is positive and the firm will not use only the clean technology. If the firm uses both technologies, the following condition must be satisfied

$$D'(q)q + D(q) = \beta = p_e e'(q_d), \tag{7}$$

that establishes the well known condition that the marginal costs must be equal. However, as the marginal cost of dirty output decreases with the polluting input price, there will exist a threshold for this price defined by condition

$$D'(q_d)q_d + D(q_d) = \beta = \bar{p}_e e'(q_d), \tag{8}$$

such that is $p_e \leq \bar{p}_e$ the firm will not invest in the clean technology. Notice that if the price is low enough, the condition $D'(q_d)q_d + D(q_d) = \bar{p}_e e'(q_d)$ will be hold for a marginal cost of the dirty output lower than β. Then, all the output is produced with the dirty technology. These results can be summarized in the following proposition

Proposition 1 *The firm will not invest in the clean technology if the polluting input price, p_e, is lower or equal to the threshold value, $\bar{p}_e$, defined by condition (8). If $\bar{p}_e < p_e$, the investment is positive, but the firm does not profitable to substitute completely the dirty technology.*

If we assume that $\bar{p}_e < p_e$, condition (7) defines implicitly the total output for the monopoly as a function of β, $q^m(\beta)$, and the dirty output as a function of β and p_e, $q_d^m(\beta, p_e)$.[7] Using again the Implicit Function Theorem we obtain the slopes of these functions[8]

$$\frac{dq^m}{d\beta} = \frac{1}{D''q + 2D'} < 0, \ \frac{\partial q_d^m}{\partial \beta} = \frac{1}{p_e e''} > 0, \ \frac{\partial q_d^m}{\partial p_e} = -\frac{e'}{p_e e''} < 0,$$

that allows us to conclude that

$$\frac{\partial q_c^m}{\partial \beta} = \frac{dq^m}{d\beta} - \frac{\partial q_d^m}{\partial \beta} < 0, \ \frac{\partial q_c^m}{\partial p_e} = -\frac{\partial q_d^m}{\partial p_e} > 0.$$

Thus, an increase in the LMC of the clean output reduces the clean output and increases the dirty output with a negative net effect on total output. On the other hand, an increase in the price of the polluting output reduces the dirty output and increases the clean output, but leaves unchanged the total output.

Moreover, using these expressions we can obtain the production-mix used by the firm defined as the ratio between dirty and total output

$$\tilde{q}^m = \frac{q_d^m(\beta, p_e)}{q^m(\beta)}. \tag{9}$$

From the previous results it is immediate that

$$\frac{\partial \tilde{q}^m}{\partial \beta} > 0, \ \frac{\partial \tilde{q}^m}{\partial p_e} < 0.$$

An increase in the LMC of the clean output increases the weight that dirty output has on total output whereas an increase in the input price has the contrary effect.

[7] The superscript m represents the outcome of the monopoly equilibrium.

[8] Notice that as the firm operates at full capacity, an increase in the price of the polluting input will increase the price of the monopoly in the short-run. Thus, the effects we obtain applying the implicit function theorem are the long-run effects that take into account the adjustment in the capacity of the clean technology.

2.2 The Efficient Outcome

The efficient outcome is given by the quantities that maximize net social welfare defined as the sum of consumers' surplus plus producer surplus minus environmental damages[9]

$$\max_{\{q_c,q_d\}} NSW = \int_0^q D(y)dy - \beta q_c - p_e e(q_d) - ED(e(q_d)).$$

The solution to this optimization problem must satisfy the following condition for an interior solution:

$$D(q) = \beta = (p_e + ED'(e(q_d)))e'(q_d). \tag{10}$$

where the last term stands for the *full marginal cost* of the dirty output, that includes the marginal cost of production plus the marginal environmental damages caused by the production of the good with the dirty technology. The efficiency requires that the price be equal to the marginal costs and hence that the marginal cost of the clean production be equal to the full marginal cost of the dirty production. As occurs for the market equilibrium, there also exists a threshold for the price of the polluting input defined by the condition $D(q_d^*) = \beta = (\bar{p}_e^* + ED'(e(q_d^*)))e'(q_d^*)$, such that if $p_e \leq \bar{p}_e^*$ it is not efficient to invest in the clean technology.[10] It is easy to show using conditions (7) and (10) that $\bar{p}_e^*$ is lower than $\bar{p}_e^m$ what implies that if p_e belongs to the the interval ($\bar{p}_e^*$, $\bar{p}_e^m$] the firm should invest in the clean technology, but it does not.

Condition (10) implicitly defines the quantities that maximize net social welfare, $q^*(\beta)$ and $q_d^*(\beta, p_e)$, and hence $q_c^*(\beta, p_e) = q^*(\beta) - q_d^*(\beta, p_e)$. Comparing the prices for the monopoly equilibrium and the efficient outcome assuming that p_e is higher than $\bar{p}_e^m$, $D(q^*) - D(q^m) = D'(q^m)q^m < 0$, we obtain the standard result that establishes that the monopoly total output is lower than the efficient level of total output.

On the other hand, we have that $ED'(e(q_d^*))e'(q_d^*) = p_e(e'(q_d^m) - e'(q_d^*)) > 0$, and we can conclude that the monopoly dirty output is larger than the efficient level of dirty output since the marginal cost of the dirty output is an increasing function. Then, as

[9] In the rest of the paper we focus on the study of the long-run decisions of the firm and state the optimization problems in terms of the clean and dirty output using the long-run production cost of the clean technology.

[10] The superscript $*$ is used to represent the efficient solution.

the efficient total output is larger than the monopoly total output and the efficient dirty output is lower, the monopoly clean output is lower than the efficient level of clean output. Consequently

$$\tilde{q}^m = \frac{q_d^m}{q^m} > \tilde{q}^* = \frac{q_d^*}{q^*}.$$

The distortions caused by the market power of the firm and the negative externality lead to a production-mix that gives more weight to the dirty output. On one hand, as the firm does not take into account the negative externality of its dirty output, this is above the efficient level. On the other hand, the market power yields a lower level of total output. Thus, the combination of these two distortion yield a monopoly production-mix that is larger than the efficient production-mix. The following proposition summarizes the results obtained for this comparison:

Proposition 2 *The monopolist's clean production is lower than the efficient level whereas for the dirty production occurs the contrary so that the monopolist's production-mix is higher than the efficient level. As expected, the monopoly total production is lower than the efficient level yielding a higher price. Moreover, there exists an interval of prices for the polluting input for which the firm should invest in the clean technology, but it does not find it profitable.*

3 Emission Taxes versus Feed-in Subsidies

In this section, we study the second-best policy considering two alternative policy instruments: a tax on emissions and a feed-in subsidy on the clean output. For characterizing the optimal policy, we calculate the *Stackelberg equilibrium* of a policy game where the regulator acts as the leader of the game. Thus, in the first stage, the regulator selects the level of the policy instrument with the aim of maximizing the net social welfare, and in the second stage, the firm chooses the levels of clean and dirty output. We will begin with the analysis of the emission tax.

3.1 The Emission Tax

Solving by backward induction, we solve first the second stage. At this stage, the monopolist, given the level of the tax selected by the regulator in the first stage, chooses the quantities that maximize its profits

$$\max_{\{q_c, q_d\}} \pi^t = D(q_c + q_d)(q_c + q_d) - \beta q_c - (p_e + t)e(q_d) + S,$$

where S stands for a lump-sum subsidy that we assume equal to the tax revenues.[11] In this way, the environmental policy is *neutral* from a fiscal point of view, i.e. it does not affect the public deficit/surplus. The aim of the regulator is not to collect money, but to reduce emissions.

The FOCs of the problem yield

$$D'(q)q + D(q) = \beta = (p_e + t)e'(q_d). \tag{11}$$

It is straightforward that total production of the monopoly does not change with the application of the tax, but the tax modify the production-mix. The RHS of this condition defines the reaction function $q_d(t)$ that has negative slope

$$\frac{dq_d}{dt} = -\frac{e'}{(p_e + t)e''} < 0, \tag{12}$$

that for a constant total output implies that $dq_c/dt = -dq_d/dt$. Although the absolute values of the slope are the same, we find that the clean output is a *strategic complement* of the tax whereas the dirty output is a *strategic substitute*. A variation in the tax affects the production-mix, but it has no effects on the total output and price.

Next, we solve the first stage. Substituting $q_c(t)$ and $q_d(t)$ in the net social welfare, we obtain an expression that depends on t

$$\max_{\{t\}} NSW = \int_0^q D(y)dy - \beta q_c(t) - p_e e(q_d(t)) - ED(e(q_d(t))).$$

As the tax does not affect total output, the regulator's optimization problem can be rewritten as follows

$$\min_{\{t\}} \left\{ \beta q_c(t) + p_e e(q_d(t)) + ED(e(q_d(t))) \right\}.$$

[11]The superscript t stands for the outcome of the market equilibrium when a tax is applied by the regulator.

The FOC for this problem is

$$\beta \frac{dq_c}{dt} + (p_e + ED')e' \frac{dq_d}{dt} = 0,$$

as $dq_c/dt = -dq_d/dt$, the previous condition yields the efficient condition $(p_e + ED')e' = \beta$ that taking into account (11) yields

$$(p_e + ED')e' = \beta = (p_e + t)e',$$

what implies that $t^* = ED'(e^*)$. Thus, the second-best emission tax is the Pigouvian tax and the monopolist implements the efficient level of dirty production, $q_d^t = q_d^* < q_d^m$ but the total output is lower, $q^t = q^m < q^*$ what implies that $q_c^m < q_c^t < q_c^*$. Then, we have that

$$\tilde{q}^m = \frac{q_d^m}{q^m} > \tilde{q}^t = \frac{q_d^*}{q^m} > \tilde{q}^* = \frac{q_d^*}{q^*}.$$

Thus, the optimal tax induces the firm to produce the efficient level of emissions, but it does not affect the total output. The result is that the clean output is larger than the level achieved by the monopoly when there is no regulation but lower than the efficient level and consequently the same occurs for the production-mix. Next proposition summarizes these results.

Proposition 3 *The second-best tax rate is the Pigouvian tax. The tax induces the monopolist to implement the efficient level of the dirty output but does not modify the total output. The result is that although the monopolist increases the clean output and reduces the production-mix, the efficient level of the clean output is not achieved.*

As the tax is equal to the marginal damage of emissions, the new marginal cost curve corresponds to the full marginal cost and the monopolist chooses the efficient level of the dirty output, but the tax does not affect the equilibrium point defined by the marginal cost of the clean technology, β. The consequence is that the total output is not affected by the emission tax. Thus, we could say that the tax only corrects the distortion causes the negative externality changing the composition of the total output, but does not affect the firm's market power. The price charged to the consumers continues being the monopoly price.

3.2 The Feed-in Subsidy

If the firm faces a feed-in subsidy, the optimization problem that yields the quantities in the second stage is

$$\max_{\{q_c,q_d\}} \pi^s = D(q_c+q_d)(q_c+q_d) - (\beta - s)q_c - p_e e(q_d) - T,$$

where s is the subsidy per unit of clean output and T stands for a lump-sum tax that we assume equal to the total subsidy.[12] Again, the idea is to define an environmental policy that is *neutral* from a fiscal point of view.

The FOCs of the problem yield the following condition

$$D'(q)q + D(q) = \beta - s = p_e e'(q_d). \tag{13}$$

The reaction functions define by these conditions, $q(s)$ and $q_d(s)$, have the following slopes

$$\frac{dq}{ds} = -\frac{1}{D''q + 2D'} > 0, \quad \frac{dq_d}{ds} = -\frac{1}{p_e e''} < 0, \tag{14}$$

resulting in a dq_c/ds that is positive.

The clean output is a *strategic complement* of the policy instrument whereas the dirty output is a *strategic substitute.* Thus, the subsidy decreases the production-mix, however, now the effect on total output is positive and the feed-in subsidy will cause a reduction in the price.

Next, we solve the first stage. Taking into account the reaction functions $q_c(s)$ and $q_d(c)$, the net social welfare is a function of s

$$\max_{\{s\}} NSW = \int_0^{q(s)} D(y)dy - \beta q_c(s) - p_e e(q_d(s)) - ED(e(q_d(s))).$$

The maximization with respect to the subsidy gives the following condition

$$(D-\beta)\frac{dq}{ds} + (\beta - (p_e + ED')e')\frac{dq_d}{ds} = 0. \tag{15}$$

This condition establishes that the variation in net social welfare caused by the increase in total output that is given by the difference between the price and the marginal cost

[12] The superscript s stands for the outcome of the market equilibrium when a feed-in subsidy is applied by the regulator.

of clean output must compensate the variation in net social welfare coming from the reduction in dirty output that depends on the difference in the marginal cost of both technologies taking into account the environmental damages in the case of the dirty production.

Using the FOCs for the maximization of profits, this condition can be rewritten as follows

$$-(s + D'q)\frac{dq}{ds} + (s - ED'e')\frac{dq_d}{ds} = 0,$$

that allows us to obtain an expression for the subsidy

$$s^* = \frac{-D'(q)q\frac{dq}{ds} - \frac{dq_d}{ds}ED'e'}{\frac{dq_c}{ds}} = \frac{\frac{p}{|\eta|}\frac{dq}{ds} - ED'e'\frac{dq_d}{ds}}{\frac{dq_c}{ds}} > 0, \tag{16}$$

where $|\eta|$ stands for the price-elasticity of demand function. The subsidy presents two components. The first one reflects the distortion caused by the market power of the firm and is inversely related to the price-elasticity of the demand function. The second component depends on the environmental damages and appears in the expression because of the distortion caused by the negative externality.

Notice that if the efficient conditions are satisfied, the FOC (15) also holds and the subsidy could implement the efficient outcome, but then the efficient outcome should satisfy that $ED'(e^*)e'(q_d^*) = s^* = -D'(q^*)q^*$. In general, this condition will not hold and that the optimal subsidy given by (16) will not implement the efficient outcome. Then, we could have two types of solutions: one when $D - \beta > 0$ what implies that $\beta - p_e e' - ED'e' > 0$ and the other when the contrary occurs. When $D(q^s) > \beta$ as $\beta = D(q^*)$ for the efficient output, we can conclude that q^s is lower than q^*.[13] On the other hand, as then $\beta > (p_e + ED')e'$ and for the efficient outcome we have that $\beta = (p_e + ED')e'$, q_d^s is lower than the q_d^* since the full marginal cost is increasing with respect to the dirty output. However, these comparisons do not allow us to get a clear sign for the comparison of the clean output and hence for the comparison of the production-mix. When $D - \beta < 0$ and $\beta - p_e e' - ED'e' > 0$, we obtain that q^s is larger than q^* and that q_d^s is also larger than q_d^*, but again the comparison of the levels of the clean output and production-mix remains undetermined.

[13] Where the supersript s stands for the equilibrium when a subsidy is applied by the regulator.

Next, we compare the outputs supported by the subsidy with the monopoly outputs. From the FOCs (7) and (13), we obtain that the difference in the marginal revenues is equal to the subsidy

$$D(q^m) + D'(q^m)q^m - (D'(q^s)q^s + D(q^s)) = s > 0.$$

As we assume that the marginal revenue is decreasing and we have shown that the subsidy is positive, we obtain that q^m is lower than q^s. Moreover, these conditions also establish that the subsidy must be equal to the difference in the marginal cost of the dirty output

$$s = p_e(e'(q_d^m) - e'(q_d^s)) > 0,$$

which allows us to conclude that q_d^s is lower than q_d^m since the marginal cost is increasing. Then, the q_c^m must be lower than q_c^s and $\tilde{q}^s$ must be also lower than $\tilde{q}^s$. Next proposition summarizes these comparisons.

Proposition 4 *The optimal subsidy leads the monopolist to reduce the dirty output and increase the total output resulting in a higher level of clean output and a lower level of the production-mix. However, the dirty and total outputs could be larger or lower than the efficient levels and the same occurs for the clean output.*

Finally, we compare the outputs resulting from the application of the second-best policies. From Props. 3 and 4, we know that the total output is not affected by the tax, whereas the subsidy increases the total output so that we can conclude that q^t is lower than q^s. However, as q_d^s can be higher or lower than q_d^* we cannot establish whether q_d^t is higher or lower than q_d^s. When q_d^t is higher than q_d^s, it is straightforward that q_c^t is lower than q_c^s and that the production-mix is higher when a tax is applied. This ambiguity in the comparison between the dirty output corresponding to the different policies does difficult to rank them in terms of net social welfare. However, when q_d^t is higher than q_d^s, that can be a plausible result, it is easy to check that if the productivity of the capital invested in the clean technology is high enough the subsidy will yield a higher net social welfare. Writing the difference in net social welfare as follows

$$NSW(t) - NSW(s) = \int_0^{q^t} D(y)dy - \beta q_c^t - p_e e(q_d^t) - ED(e(q_d^t))$$
$$- \left(\int_0^{q^s} D(y)dy - \beta q_c^s - p_e e(q_d^s) - ED(e(q_d^s)) \right)$$
$$= \int_0^{q^t} D(y)dy - \int_0^{q^s} D(y)dy + \beta(q_c^s - q_c^t) + p_e(e(q_d^s) - e(q_d^t)) + ED(e(q_d^s)) - ED(e(q_d^t)),$$

we have that all terms are negative when $q_d^t > q_d^s$ and $q^s > q^t$ except the second one that is positive because $q_c^s > q_c^t$, but this component depends on β the marginal cost of the clean technology. For this reason, we expect a positive value for this expression when β is low enough. In the next section, we give more structure to our model and investigate whether this hypothesis is satisfied.

4 The Linear-quadratic Case

In this section we consider a monopoly that faces a market demand represented by a linear demand function $p = A - q$, where q is the firm's output. The firm can produce the output using a *dirty* technology that employs a polluting factor given by $q_d = a_d e^{1/2}$, where e stands for the polluting input and a_d is a positive parameter measuring factor productivity. After an appropriate choice of measurement units we can say that each unit of input generates one unit of pollution. According to this technology, the production cost of the dirty output is $C_d(q_d) = (p_e/\alpha)q_d^2$, where p_e is the input market price and $\alpha = a_d^2$. Pollution generates environmental damages given by the function $D(e) = de,\ d > 0$. Alternatively, the firm can also produce the same good using a *clean* technology that operates with constant returns $q_c = a_c k$, and can be bought at a cost equal to p_k. Thus, the production cost of the clean technology is $C_c(q_c) = \beta q_c$, where β is given by p_k/a_c. Total output of the firm is given by $q = q_c + q_d$ and we assume that $\beta < A$.[14]

[14]Our specification gives the same cost structure for the polluting firms that the one used by Fischer et al. (2018) in their analysis of the subsidies with renewable energy standards, although in their model the market for the polluting firms is competitive.

For this specification of the model the monopoly equilibrium quantities for the clean and dirty outputs are given by the following expressions

$$q_c^m = \frac{(A-\beta)p_e - \alpha\beta}{2p_e}, \quad q_d^m = \frac{\alpha\beta}{2p_e}, \tag{17}$$

so that the total output is $q^m = (A-\beta)/2$. The first expression in (17) clearly establishes that the firm will only produce a clean output if the price of the polluting input is not too low. In particular if

$$p_e > \bar{p}_e^m = \frac{\alpha\beta}{A-\beta}, \tag{18}$$

as was established in Section 2.1. Moreover, using the quantities in (17) we can obtain the production-mix used by the firm defined as the ratio between dirty and total output

$$\tilde{q}^m = \frac{q_d^m}{q^m} = \frac{\alpha\beta}{(A-\beta)p_e - \alpha\beta}. \tag{19}$$

For these quantities, the net social welfare and profits are

$$NSW^m = \frac{3(A-\beta)^2 p_e^2 + 2\alpha\beta^2(p_e - d)}{8p_e^2}, \quad \pi^m = \frac{(A-\beta)^2 p_e + \alpha\beta^2}{4p_e}. \tag{20}$$

On the other hand, the quantities that maximize net social welfare are

$$q_c^* = \frac{2(A-\beta)(d+p_e) - \alpha\beta}{2(p_e+d)} \quad q_d^* = \frac{\alpha\beta}{2(p_e+d)}. \tag{21}$$

Adding these quantities, we obtain the total output, $q^* = A - \beta$. Notice that to have a positive clean production a high enough price is also needed. However, it is easy to check that if condition (18) holds, the efficient level of the clean output is also positive because $\bar{p}_e^m > \bar{p}_e^*$. The production-mix corresponding to the efficient outcome is

$$\tilde{q}^* = \frac{\alpha\beta}{2(A-\beta)(d+p_e) - \alpha\beta}. \tag{22}$$

4.1 The Emission Tax

When a tax is used to control emissions, the tax rate that maximizes the net social welfare is: $t^* = d$, and the clean and dirty outputs are

$$q_c^t = \frac{(A-\beta)(d+p_e) - \alpha\beta}{2(d+p_e)}, \quad q_d^t = q^* = \frac{\alpha\beta}{2(d+p_e)}, \tag{23}$$

where the clean output is positive if the condition (18) is satisfied. These two quantities gives the following production-mix

$$\tilde{q}^t = \frac{\alpha\beta}{(A-\beta)(d+p_e)-\alpha\beta}. \tag{24}$$

Fig. 1 graphically illustrates the effects of the Pigouvian tax on the monopoly equilibrium.

$\Rightarrow$ FIG. 1 $\Leftarrow$

In the figure, we see that the tax increases the marginal cost of the dirty output. As the tax is equal to the marginal damage of emissions, the new marginal cost curve corresponds to the full marginal cost and the monopolist chooses the efficient level of the dirty output, but the tax does not affect the equilibrium point defined by the marginal cost of the clean technology, $MC_c = \beta$. The result is that the total output is not affected by the emission tax. Thus, we could say that the tax only corrects the distortion causes the negative externality changing the composition of the total output, but does not affect the firm's market power.

Finally, we calculate the players' payoffs

$$NSW^t = \frac{3(A-\beta)^2}{8} + \frac{\alpha\beta^2}{4(p_e+d)}, \quad \pi^t = \frac{(A-\beta)^2}{4} + \frac{\alpha\beta^2}{4(p_e+d)}. \tag{25}$$

Usually, a second-best policy improves welfare with respect to the market equilibrium without regulation, but we want to check this point for our model. Comparing the payoffs presenting above with those obtained for the monopoly equilibrium without regulation we obtain the following signs

$$\begin{aligned} NSW^t - NSW^m &= \frac{\alpha\beta^2 d^2}{4(p_e+d)p_e^2} > 0, \\ \pi^t - \pi^m &= -\frac{\alpha\beta^2 d^2}{4(p_e+d)^2 p_e} < 0, \end{aligned}$$

and the conclusion is that

Proposition 5 *The Pigouvian tax is welfare improving, although it reduces the monopolist's profit.*

In Fig. 1, the area B represents the increase in the production cost because of the reduction in the dirty output. This causes the decrease in the monopoly profit. On the other hand, the area $B+E$ reflects the reduction in damages. Consequently the area E stands for the welfare gains.

4.2 The Feed-in Subsidy

When a subsidy is applied to regulate the monopoly emissions, the subsidy that maximizes net social welfare is

$$s = \frac{(A-\beta)p_e^2 + 2\alpha\beta d}{p_e^2 + 2\alpha(d+p_e)}. \tag{26}$$

The subsidy is positive but the net marginal cost of the clean technology, $\beta - s$, could be negative

$$\beta - s = \frac{(2\alpha\beta - (A-2\beta)p_e)p_e}{p_e^2 + 2\alpha(d+p_e)}.$$

This expression says us that if the marginal cost of the clean technology is high enough, in particular, if $\beta \geq A/2$, the net subsidy cannot be negative. However, in this paper we are interested in the contrary case, in a clean technology that is efficient enough to yield a low marginal cost and consequently a large market size for the output. For this reason, we assume from now that $\beta < A/2$ and that the price of the polluting input is not too high

$$\frac{2\alpha\beta}{A-2\beta} > p_e. \tag{27}$$

Notice that this upper bound is compatible with the lower bound defined by (18). Moreover, if this condition is satisfied we can conclude that

$$\frac{\partial s}{\partial d} = \frac{2\alpha p_e(2\alpha\beta - (A-2\beta)p_e)}{(p_e^2 + 2\alpha(d+p_e))^2} > 0.$$

An increase in the marginal damages brings an augmentation in the subsidy.[15] For a feed-in subsidy, the clean and dirty outputs are

$$q_c^s = \frac{2(\alpha+p_e)((A-\beta)p_e - \alpha\beta) + \alpha A(2d+p_e)}{2(p_e^2 + 2\alpha(d+p_e))}, \quad q_d^s = \frac{\alpha(2\alpha\beta - (A-2\beta)p_e)}{2(p_e^2 + 2\alpha(d+p_e))}. \tag{28}$$

[15] For the rest of parameters, the effect is ambiguous.

Adding these two quantities the total output is obtained

$$q^s = \frac{p_e((A-\beta)p_e - \alpha\beta) + \alpha A(p_e + d)}{p_e^2 + 2\alpha(p_e + d)}. \tag{29}$$

The clean output is positive if condition (18) holds whereas the dirty output is positive if condition (27) is satisfied. With a positive net marginal cost of the clean technology we expect a positive dirty output because the marginal costs of both technologies must be the same in the equilibrium. Thus, we assume that the price of the polluting input is large enough to guarantee that the clean output is positive for all the cases studied in the paper, but not to high in order to guarantee a positive dirty output when the regulator applies a feed-in subsidy.[16]

The quantities in (28) also allow us to calculate the production-mix

$$\tilde{q}^s = \frac{\alpha(2\alpha\beta - (A-2\beta)p_e)}{2(\alpha + p_e)((A-\beta)p_e - \alpha\beta) + \alpha A(2d + p_e)}. \tag{30}$$

Finally, we calculate the net social welfare and the firm's profits.

$$NSW^s = \frac{A(3A-4\beta)}{8} + \frac{(2(\alpha+p_e)\beta - Ap_e)^2}{8(p_e^2 + 2\alpha(p_e+d))}, \tag{31}$$

$$\pi^s = \frac{A^2}{4} + \frac{(2(\alpha+p_e)\beta - Ap_e)^2(p_e+d)p_e}{4(p_e^2 + 2\alpha(p_e+d))^2} - \frac{A(2(\alpha+p_e)\beta - Ap_e)p_e}{2(p_e^2 + 2\alpha(p_e+d))}. \tag{32}$$

Once calculated the quantities induced by the application of the feed-in subsidy, it is interesting to compare them with those obtained for the monopoly equilibrium and efficient solution. The differences with respect to the monopoly equilibrium outputs are given by the following expressions:

$$\begin{aligned}
q_c^s - q_c^m &= \frac{(p_e+\alpha)\left((A-\beta)p_e^2 + 2d\alpha\beta\right)}{2p_e(p_e^2 + 2\alpha(p_e+d))} > 0, \\
q_d^s - q_d^m &= -\frac{\alpha((A-\beta)p_e^2 + 2\alpha\beta d)}{2p_e(p_e^2 + 2\alpha(p_e+d))} < 0, \\
q^s - q^m &= \frac{(A-\beta)p_e^2 + 2\alpha\beta d}{2(p_e^2 + 2\alpha(p_e+d))} > 0.
\end{aligned}$$

[16]For a polluting input price higher than the upper bound defined by (27), the model gives a corner solution with $q_d^s = 0$. In this paper, we focus on the interior solution assuming the polluting input price is not so high as to yield a complete substitution of the dirty technology by the clean technology.

As was established in Prop. 3, the feed-in subsidy decreases the dirty output but increases the clean output for a larger quantity resulting in an increase of the total output. As the dirty output decreases and the contrary occurs for the clean output, the production-mix decreases.

The comparison with the efficient quantities yields

$$\begin{aligned} q_c^s - q_c^* &= -\frac{\alpha\left(2d+p_e\right)\left((A-\beta)p_e+(A-2\beta)d\right)}{2(p_e^2+2\alpha(d+p_e))(d+p_e)} < 0, \\ q_d^s - q_d^* &= -\frac{\alpha p_e((A-\beta)p_e+(A-2\beta)d)}{2(p_e^2+2\alpha(d+p_e))(d+p_e)} < 0, \\ q^s - q^* &= -\frac{\alpha((A-\beta)p_e+(A-2\beta)d)}{p_e^2+2\alpha(d+p_e)} < 0, \end{aligned}$$

$$\tilde{q}^s - \tilde{q}^* = -\frac{2\alpha\left((A-\beta)p-\alpha\beta\right)\left((A-2\beta)d+(A-\beta)p\right)}{(2(\alpha+p_e)((A-\beta)p_e-\alpha\beta)+\alpha A(2d+p_e))(2(A-\beta)(d+p_e)-\alpha\beta)} < 0.$$

The following proposition summarizes these results and Fig. 2 shows them

Proposition 6 *The optimal feed-in subsidy given by (26) leads to the monopolist to increase the clean output and to reduce the dirty output resulting in an increase of the production-mix and total output. However, if $\beta \leq A/2$ all the outputs are bellow the efficient levels and the same occurs for the production-mix.*[17]

⇒ FIG. 2 ⇐

In the graph, the reduction in the marginal cost caused by the subsidy increases the firm's total output from q^m to q^s, but it does not reach the efficient level given by q^*. On the other hand, the dirty output decreases until q_d^s, a level that is bellow the efficient dirty output level. The distance between q^s and q_d^s that represents the clean output is clearly lower than the distance between q^* and q_d^*.

Next, we evaluate the effect that the subsidy has on the players' payoffs

$$\begin{aligned} NSW^s - NSW^m &= \frac{((A-\beta)p_e^2+2\alpha\beta d)^2}{8p_e^2(p_e^2+2\alpha(p_e+d))} > 0, \\ \pi^s - \pi^m &= -\frac{(p_e+\alpha)((A-\beta)p_e^2+2\alpha\beta d)^2}{4p_e(p_e^2+2\alpha(p_e+d))^2} < 0, \end{aligned}$$

[17] If $\beta > A/2$ the dirty and clean outputs could be larger or lower than the efficient levels and the same occurs for the total output as was established in Prop. 4.

so that we can conclude that

Proposition 7 *A feed-in subsidy is welfare improving, although it reduces the monopolist's profits.*

In Fig. 2, the area $B+F+G+K+L$ represents the increase in the production cost. $B+F+G$ is the increase in costs because $q_d^m - q_d^s$ is produced at a higher cost whereas the increase in costs given by $K+L$ is the result of expanding the output. On the other hand, as the total output increases, the revenues increase by the area $J+K+L$ but decreases by the area H. Thus, $J-H-B-F-G$ stands for the drop in profit. Moreover, damages decrease by the areas $B+E+G$ and the net consumers' surplus increases by the area $H+I$. The net effect is an increase in the net social welfare equal to the area $I+J-F+E$.

4.3 Comparing the outcome of both policies

The analysis developed in the previous sections establishes that the two policy instruments are welfare improving. In this section, we compare them with the aim of finding out if one of them yields a larger welfare than the other. We begin this comparative analysis, comparing the outputs

$$\begin{aligned}
q_c^t - q_c^s &= -\frac{(A-\beta)(\alpha+p_e+d)p_e^2 + (Ap_e+2\beta d)\alpha d}{2(p_e^2+2\alpha(p_e+d))(p_e+d)} < 0, \\
q_d^t - q_d^s &= \frac{\alpha p_e((A-\beta)p_e + (A-2\beta)d)}{2(p_e+d)(p_e^2+2\alpha(p_e+d))} > 0, \\
q^t - q^s &= -\frac{(A-\beta)p_e^2 + 2\alpha\beta d}{2(p_e^2+2\alpha(p_e+d))} < 0.
\end{aligned}$$

As the subsidy gives a higher level of the clean output and a lower level of the dirty output, the production-mix is higher when a tax is applied that when a subsidy is used to promote the use of the clean technology. The next proposition summarizes these results and Fig. 2 shows these relationships

Proposition 8 *The feed-in subsidy yields a higher clean output and a higher total output resulting in a lower production-mix. Moreover, if $\beta \leq A/2$ the dirty output is lower when a subsidy is applied to promote clean output.*

The tax yields a higher level of the dirty output but a lower level of the total output, the result is that the distance representing the clean output for the tax is lower than the distance representing the clean output for the feed-in subsidy.

Next, we compare the net social welfare and the profits of the firm[18]

$$
\begin{aligned}
NSW^t - NSW^s &= -\frac{2\alpha\beta d^2(2A-3\beta) + dp_e(A-\beta)(4\alpha\beta + (A-\beta)p_e) + p_e^3(A-\beta)^2}{8(p_e^2 + 2\alpha(p_e+d))(p_e+d)} < 0, \\
\pi^t - \pi^s &= -\frac{\alpha\beta^2 d^2}{4(p_e+d)^2 p_e} + \frac{(p_e+\alpha)((A-\beta)p_e^2 + 2\alpha\beta d)^2}{4p_e(p_e^2 + 2\alpha(p_e+d))^2} > 0.
\end{aligned}
$$

Proposition 9 *A feed-in subsidy leads to a larger net social welfare but to lower profits for the firm provided that* $\beta \leq A/2$.

We can compare graphically the players' payoffs using the Fig. 2. In this figure, the area B stands for the reduction in the firm's profits when a tax is applied whereas the area $H - J + B + F + G$ represents the reduction in profits when a feed-subsidy is used to promote the clean technology. Our analysis establishes that $H - J + F + G$ is positive what means that the feed-subsidy causes a reduction in the firm's profit higher than that originated by the tax. With a subsidy the production cost increases more that with a tax and there is also a variation in the profit given by the area $H - J$ because the increase in the total output. On the other hand, the area E represents the increase in net social welfare for a tax, and the area $I + J - F + E$ the increase in net social welfare when a feed-subsidy is used. The last proposition says us that $I + J - F$ is positive. The subsidy yields an increase in net social welfare larger than that provoked by the tax. With a tax, the increase in welfare comes only from the reduction in damages. Instead, for a subsidy we have to add to the reduction in damages an increase in the consumers' surplus represented by the area I along with an increase in the profit given by the area J because the firm sells a larger quantity of the commodity. These positive variations are larger than the increase in costs that is is not compensated by the reduction in damages represented by the area F that is caused by the reduction in the dirty output.

[18] The sign in the difference in profits is determined in the Appendix.

5 Emissions Taxes with Feed-in Subsidies

In this last section, we characterize the optimal policy when the regulator uses the two policy instruments studied above. When this occurs, the maximization of net profits is given by the following expression:

$$\max_{\{q_c, q_d\}} \pi = D(q_c + q_d)(q_c + q_d) - (\beta - s)q_c - (p_e + t)e(q_d) + M,$$

where M is a lump-sum transfer that we assume equal to the difference $T - S = te(q_d) - sq_c$.

Solving first the second stage, we calculate first the FOCs for the maximization of profits

$$\frac{\partial \pi}{\partial q_c} = D'(q)q + D(q) - \beta + s = 0, \tag{33}$$

$$\frac{\partial \pi}{\partial q_d} = D'(q)q + D(q) - (p_e + t)e'(q_d) = 0. \tag{34}$$

From these conditions we obtain that the dirty output is given by the following expression $\beta - s = (p_e + t)e'(q_d)$ that implicitly defines the monopolist's reaction function for the dirty output: $q_d(s,t)$. It is easy to check that the slopes of the reaction function are negative

$$\frac{\partial q_d}{\partial s} = -\frac{1}{(p_e + t)e''} < 0, \frac{\partial q_d}{\partial t} = -\frac{e'}{(p_e + t)e''} < 0,$$

and we can conclude that the dirty output is a *strategic substitute* of both the subsidy and the tax as occurs when only a policy instrument is used. In fact, $\partial q_d/\partial t$ coincides with (12). On the other hand, condition (33) establishes that the total output only depends on the subsidy as occurs in Subsection 3.2. Finally, the reaction function for the dirty output can be calculated as the difference between the total output and the dirty output: $q_c(s,t) = q(s) - q_d(s,t)$, so that the slopes are

$$\frac{\partial q_c}{\partial s} = \frac{dq}{ds} - \frac{\partial q_d}{\partial s} = -\frac{1}{D''q + 2D'} + \frac{1}{(p_e + t)e''} > 0,$$

where the first derivative is given by (14), and $\partial q_c/\partial t = -\partial q_d/\partial t > 0$. Again, as occurs when only a policy instrument is used, the clean output is a *strategic complement* of both the the subsidy and the tax.

Next, we move to the first stage. Substituting $q_c(s,t)$ and $q_d(s,t)$ in the net social welfare, we obtain an expression that depends on the policy instruments

$$\max_{\{s,t\}} NSW = \int_0^{q(s)} D(y)dy - \beta q_c(s,t) - p_e e(q_d(s,t)) - ED(e(q_d(s,t)))$$

$$= \int_0^{q(s)} D(y)dy - \beta q(s) + \beta q_d(s,t) - p_e e(q_d(s,t)) - ED(e(q_d(s,t))).$$

The FOCs for this problem are

$$\frac{\partial NSW}{\partial s} = (D-\beta)\frac{dq}{ds} + (\beta - (p_e + ED')e')\frac{\partial q_d}{\partial s} = 0, \quad (35)$$

$$\frac{\partial NSW}{\partial t} = (\beta - (p_e + ED')e')\frac{\partial q_d}{\partial t} = 0. \quad (36)$$

If the FOCs that characterize the efficient outcome are satisfied, then $D(q^*) = \beta$ and (33) yields that $s^* = -D(q^*)q^*$. The optimal feed-in subsidy is equal to the difference between the price and the marginal revenue. Moreover, taking into account that $D(q^*) = \beta$, condition (34) can be written as follows

$$\beta = -D'(q^*)q^* + (p_e + t^*)e'(q_d^*),$$

that along with condition (36) yields

$$-D'(q^*)q^* + (p_e + t^*)e'(q_d^*) = (p_e + ED'(e^*))e'(q_d^*),$$

that allows us to obtain the following expression for the optimal tax

$$t^* = ED'(e^*) + \frac{D'(q^*)q^*}{e'(q_d^*)} = ED'(e^*) - \frac{p}{|\eta|}\frac{1}{e'(q_d^*)}, \quad (37)$$

where $|\eta|$ is the price-elasticity of demand. Thus, we can conclude that

Proposition 10 *The efficient solution can be implemented through the market mechanism using two policy instruments: a emission tax on emissions lower than the environmental damages and a feed-in subsidy equal to the difference between the price and the marginal revenue.*

The optimal tax is lower than the environmental damage because the feed-in subsidy only applies to the clean output. With a subsidy on total output condition (34) would be

$$\frac{\partial \pi}{\partial q_d} = D'(q)q + D(q) - (p_e + t)e'(q_d) + s = 0,$$

and then the level of the dirty output selected by the monopolist would not depend on the subsidy and the optimal tax would be the Pigouvian tax. The tax would correct the market distortion caused by the pollution and the subsidy the market distortion caused by the firm's market power. However, if the subsidy applies only to the clean output, the tax has to correct not only the negative externality caused by the dirty output, but also the distortion originated by the market power on the dirty output. Then the tax must be lower than the environmental damage, but now as the tax is applied along with a subsidy on clean output, the result is that the efficient outcome is implemented by the regulated market equilibrium. Thus, if the subsidy can be only used to promote the clean output, the first-best tax is not anymore the Pigouvian tax as occurs when the subsidy is applied on total output.

6 Conclusions

This paper studies the use of emission taxes and feed-in subsidies for the regulation of a polluting monopoly. We study a monopoly that can produce the same good with a technology that employs a polluting input and an alternative clean technology. In the first part of the paper, we characterize the market equilibrium and the efficient outcome and highlight the important role that the input price has on the investment in the clean technology. In the second part, we calculate the second-best tax and subsidy solving a two-stage policy game between the regulator and the monopolist with the regulator acting as the leader of the game. We find that the second-best tax rate is the Pigouvian tax. The tax induces the monopolist to implement the efficient level of the dirty output but does not modify the total output. Consequently, the efficient level of the clean output is not achieved. On the other hand, the optimal subsidy leads the monopolist to reduce the dirty output and increase the total output resulting in a higher level of clean output. However, the dirty and total outputs could be larger or lower than the efficient levels and the same occurs for the clean output. This leaves the comparison between the two equilibria of the policy game undetermined, except for the total output. Nevertheless, if the dirty output is higher when a tax is applied the feed-in subsidy would yield a higher net social welfare provided that the marginal cost of the clean output is not very high. We confirm this hypothesis for a linear-quadratic model. The subsidy leads to a larger net social welfare because it increases total output whereas the tax leaves the total output unaltered. Finally, we find that the combination of an emission tax with a feed-in subsidy induces the firm to choose the efficient outputs, but the first-best tax must be lower than the Pigouvian tax. As the subsidy only applies on clean output, the tax has to correct the distortion caused by the externality and the contraction caused the firm's market power on the dirty output. Thus, our findings support the idea that feed-in subsidies open the possibility for improving the regulation of a polluting firm with market power. On one hand, they are a good alternative to taxation because they could yield a larger net social welfare. On the other hand, the combination of an emission tax and a feed-in subsidy induces the firm to implement the efficient outcome.

The analysis developed in this paper could be extended in different directions. Our analysis has focused on the study of feed-in premiums, a policy that consists of setting up a subsidy (a premium) on the output price to discriminate between dirty and clean outputs. However, several countries in Europe instead have applied a feed-in tariff that implies a direct regulation of the price for the clean output. Thus, it would be interesting to extend the analysis to consider this alternative support scheme for the clean output. On the other hand, as the research has been confined to the case of a polluting monopoly, it would be also interesting to look at other market structures to check the robustness of the results obtained in the paper. We expect that the combination of an emission tax and a feed-in subsidy works for markets with imperfect competition as the results obtained by Reichenbach and Requate (2012) for a polluting oligopoly with a clean competitive fringe suggest. A first step in this direction could be to complete the analysis of the market equilibrium developed by Fehr and Ropenus (2017) for a dominant firm with a competitive fringe calculating the optimal policy. Finally, the study of the effects that different policy instruments have on green innovation is also in the research agenda for addressing in the future.

APPENDIX

A. The sign of the difference in profits

Let's suppose that the difference is negative or zero

$$-\frac{\alpha\beta^2 d^2}{4(p_e+d)^2 p_e}+\frac{(p_e+\alpha)((A-\beta)p_e^2+2\alpha\beta d)^2}{4p_e(p_e^2+2\alpha(p_e+d))^2}\leq 0.$$

Then, it must satisfy that

$$(p_e+\alpha)((A-\beta)p_e^2+2\alpha\beta d)^2(p_e+d)^2\leq \alpha\beta^2 d^2(p_e^2+2\alpha(p_e+d))^2.$$

Developing both sides of the inequality, we obtain the following expression

$$dp\alpha^2\beta\left(4p^3(A-\beta)+4d^3\beta+dp^2(8A-5\beta)+4Ad^2p\right)$$

$$+p^3\alpha\left(4d^3\beta(A-\beta)+p^3\beta^2+Ap^3(A-2\beta)+2dp^2(A^2-\beta^2)+A^2d^2p+2d^2p\beta(3A-4\beta)\right)$$

$$+p^5\left(d+p\right)^2\left(A-\beta\right)^2>0,$$

that is strictly positive for $\beta<A/2$. Thus, we have a contradiction and we can conclude that π^t is larger than π^s.

References

[1] Antoniou, Fabio, and Roland Strausz (2017). "Feed-in Subsidies, Taxation, and Inefficient Entry." *Environmental and Resource Economics*, 67, 925-940.

[2] Barnett, A.R. (1980). "The Pigouvian Tax Rule under Monopoly." *American Economic Review*, 70, 1037-1041.

[3] Baumann, Florian, and Tim Friehe (2017). "Design Standards and Technology Adoption: Welfare Effects of Increasing Environmental Fines when the Number of Firms is Endogenous." *Environmental Economics and Policy Studies* 19, 427-450.

[4] Besanko, David (1987). "Performance versus Design Standards in the Regulation of Pollution." *Journal of Public Economics* 34, 19-44.

[5] Buchanan, J.M. (1969). "External diseconomies, corrective taxes, and market structure." *American Economic Review* 59, 174-177.

[6] Canton, Joan, Antoine Soubeyran, and Hubert Stahn (2008). "Environmental Taxation and Vertical Cournot Oligopolies: How Eco-industries Matter." *Environmental and Resource Economics* 40, 369-382.

[7] Coria, Jessica (2009). "Taxes, Permits, and the Diffusion of a New Technology." *Resource and Energy Economics* 31, 249-271.

[8] Council of European Energy Regulators (CEER) (2018). *Status Review of Renewable Support Schemes in Europe for 2016 and 2017.* C18-SD-63-03. Retrieved Apr 2019. http://www.ceer.eu/list-of-publications.

[9] Damania, D. (1996). "Pollution Taxes and Pollution Abatement in an Oligopoly Supergame." *Journal of Environmental Economics and Management* 30, 323-336.

[10] del Río, Pablo, and Miguel A. Gual (2007). "An Integrated Assessment of the Feed-in Tariff System in Spain." *Energy Policy* 35, 994-1012.

[11] Farzin, Y.H. (2003). "The Effects of Emissions Standards on Industry." *Journal of Regulatory Economics* 24, 315-327.

[12] Fehr, Nils-Henrik M. von der, and Stephanie Ropenus (2017). "Renewable Energy Policy Instruments and Market Power." *Scandinavian Journal of Economics* 119, 312-345.

[13] Fischer, Carolyn, Mads Greaker, and Knut Einar Rosendahl (2018). "Strategic Technology Policy as a Supplement to Energy Standards." *Resource and Energy Economics*, 51, 84-98.

[14] Gil-Moltó, María José, and Dimitrios Varvarigos (2013). "Emission Taxes and the Adoption of Cleaner Technologies: The Case of Environmentally Conscious Consumers." *Resource and Energy Economics* 35, 486-504.

[15] Innes, Robert, and Joseph J. Bial (2002). "Inducing Innovation in the Environmental Technology of Oligopolistic Firms." *Journal of Industrial Economics* 50, 265-287.

[16] Katsoulacos, Yannis, and Anastasios Xepapadeas (1995). "Environmental Policy under Oligopoly Endogenous Market Structure." *Scandinavian Journal of Economics* 97, 411-420.

[17] Krass, Dmitry, Timur Nedoorezov, and Anton Ovchinnikov (2013). "Environmental Taxes and the Choice of Green Technology." *Production and Operations Management* 22, 1035-1055.

[18] Martín-Herrán, Guiomar, and Santiago J. Rubio (2018). "Secon-best Taxation for a Polluting Monopoly with Abatement Investment." *Energy Economics* 73, 178-193.

[19] Moner-Colonques, Rafael, and Santiago J. Rubio (2016). "The Strategic Use of Innovation to Influence Environmental Policy: Taxes versus Standards." *BE Journal of Economic Analysis and Policy* 16, 973-1000.

[20] Montero, Juan-Pablo (2002). "Permits, Standards, and Technology Innovation." *Journal of Environmental Economics and Management* 44, 23-44.

[21] Petrakis, Emmanuel, and Anastasios Xepapadeas (2003). "Location Decisions of a Polluting Firm and the Time Consistency of Environmental Policy." *Resource and Energy Economics* 25, 197-214.

[22] Poyago-Theotoky, J.A. (2007). "The Organization of R&D and Environmental Policy." *Journal of Economic Behavior & Organization* 62, 63-75.

[23] Poyago-Theotoky, J.A. (2010). "Corrigendum to 'The Organization of R&D en Environmental Policy [J. Econ. Behav. Org. 21(1) 2007:63-75]." *Journal of Economic Behavior & Organization* 76, 449.

[24] Puller, Steven L. (2006). "The Strategic Use of Innovation to Influence Regulatory Standards." *Journal of Environmental Economics and Management* 52, 690-706.

[25] Reichenbach, Johanna, and Till Requate (2012). "Subsidies for Renewable Energies in the Presence of Learning Effects and Market Power." *Resources and Energy Economics* 34, 236-254.

[26] Requate, Till (2015). "Green Tradable Certificates versus Feed-in Tariffs in the Promotion of Renewable Energy Shares." *Environmental Economics and Policy Studies* 17, 211-239.

[27] Schmalensee, Richard (2012). "Evaluating Policies to Increase Electricity Generating from Renewable Energy." *Review of Environmental Economics and Policy* 6, 45-64.

[28] Sun, Peng, and Pu-yan Nie (2015). "A Comparative Study of Feed-in Tariff and Renewable Portfolio Standard Policy in Renewable Energy Industry." *Renewable Energy* 74, 255-262.

[29] Tamás, Mészáros Mátyás, S.O. Bade Shrestha, and Huizhong Zhou (2010). "Feed-in Tariff and Tradable Green Certificate in Oligopoly." *Energy Policy* 38, 4040-4047.

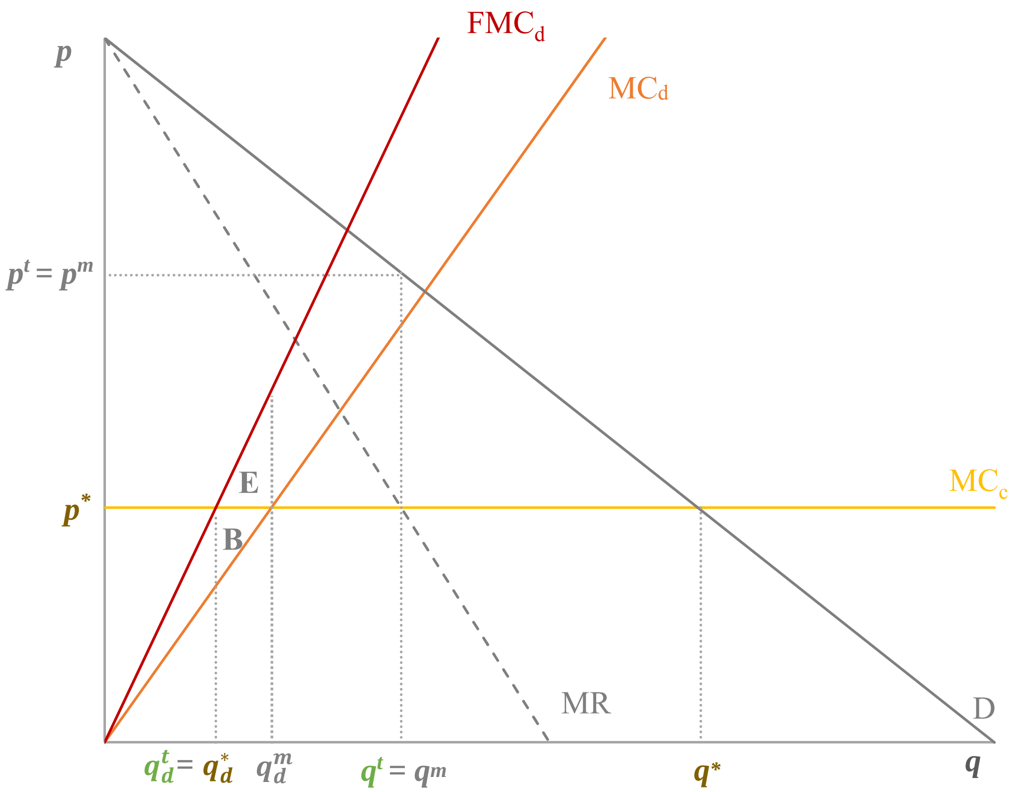

Figure 1. Effects of a Pigouvian Tax.

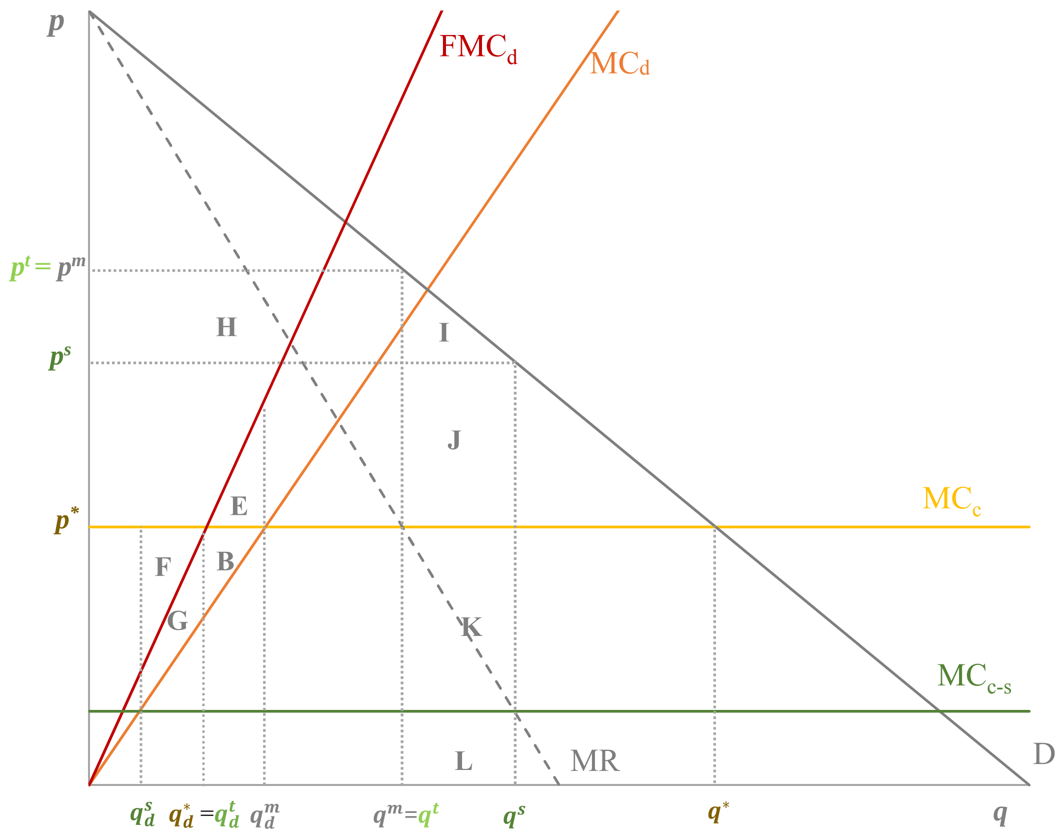

Figure 2. Effects of a Feed-in Subsidy.