Cooperation in a dynamic setting with asymmetric environmental valuation and responsibility

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Abstract

When an environmental agreement between two countries is regarded from a dynamic perspective, very often cooperation does not imply an immediate reward. More to the contrary, an agreement to reduce the emissions of pollutants is usually associated with lower flows of production income. However in a profitable agreement the current costs are more than compensated by a future cleaner environment. While this is true globally(for the two countries), neither the costs from lower emissions nor the value of a cleaner environment need to be identical for the two parts. Because the uneven benefits from cooperation are delayed, it is the effort associated with compliance what needs to be distributed between the signing countries. This paper analyzes a sharing mechanism satisfying two main properties. First, a benefit-pay-principle: the greater the benefit from cooperation the greater the relative contribution. And secondly, assuming that the responsibility from the initial environmental problem is not even across countries, a polluter's pay principle axiom requires that a country's relative contribution increases with its responsibility. Moreover, the sharing scheme must be defined to guarantee time consistency. At any intermediate instant of time, no country can do better by deviating from cooperation with the sharing mechanism presented in the paper.

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1 Introduction

In this paper we propose a distribution scheme which specifies how to share the effort that the cooperative parts in an environmental agreement need to undertake in order to mitigate an environmental problem. The sharing scheme is designed for a finite time environmental agreement and satisfies three desirable properties: time consistency, a benefits pay principle and a polluters pay principle.

We assume that two regions find it beneficial to cooperate for a finite period and reduce emissions in order to fight a common environmental problem, in particular, a stock pollution problem. One can think on a temporal agreement to reduce the emissions of carbon dioxide to the atmosphere with the aim to ameliorate the future harmful effects caused by global warming. An alternatively example could involve two neighboring countries sharing a lake polluted from wastewater discharges. An agreement to reduce discharges by the two countries across a given period might improve the water quality in the future. In both these examples, the two regions share the costs of lower emissions and the subsequent losses in production, within the cooperative period. The benefits from the agreement come at the end of the cooperative period in the form of a cleaner environment. And more importantly, we assume that the two countries differently value the environment and hence obtain different gains from cooperation. This is clearly the case in the global-warming example, as northern colder countries will be much less strongly hit by global warming than warmer agricultural and coastal regions. Likewise, in the two counties sharing a lake, one might be interested in fishing activities, while the other might give the lake a recreational value.

Typically, different gains from cooperation are not the only source of asymmetry. In stock pollution problem it is commonly the case that regions are differently responsible for the current state of the environmental problem. In the global-warming example, industrialized countries are more responsible than developing countries from the current carbon dioxide concentration in the atmosphere.[1]

[1] For the period 1850-2010, Ward and Mahowald (2014) estimate that the responsibility for a rise in temperature assigned to annex I countries can be around 58%, (42% for non-annex I). Similarly, for the period 1850-2005, using the Commuity Earth System Model (CESM), Wei *et al.* (2016) estimate

Within this framework, we consider two countries for which it is mutually beneficial to cooperate within a given period in order to reduce emissions, although neither the benefits nor the responsibility from the environmental problem at the beginning of the cooperative period are symmetric. Interestingly, the benefits from cooperation come at the end of the cooperative period in the form of a less polluted environment from this moment on. This characteristic makes our analysis different from the literature. The standard approach to distribute payoffs in a cooperative dynamic game seeks to distribute the gains from cooperation following a particular solution concept as, for example, the Shapley value, the egalitarian rule or the Nash bargaining solution (see Zaccour 2008, or Yeung and Petrosyan 2018 and references therein). We differ from this approach in two respects: firts, we do not distribute gains from cooperation but the effort that the agreement imposes on the cooperating agents, and second, we do not borrow a solution concept from the literature, but define a general distribution scheme satisfying desirable properties. The question of how to distribute the cooperative effort when the benefits come once cooperation halts what analyzed in Cabo and Tidball (2018) who proposed a time-consistent imputation distribution procedure (IDP). Based on this procedure, we define here an IDP which besides time consistency, also satisfy two properties that we consider these type of agreement should satisfy. First, a polluters pay principle, stated as: the relative total effort that a region contributes to the agreement must be positively correlated to its relative responsibility from all past emissions (i.e. to the relative responsibility from the current state of the environmental problem). Second, a benefits pay principle, stated as: the relative effort that a region contributes to the agreement must be positively correlated to the relative benefit it gets from a less polluted environment.

In Section 2 we present the model which describes cooperation and define the three desired axioms for our distribution procedure. We introduce the IDP and prove that it satisfies the three required properties. The procedure is applies to a numerical example in Section 3. Section 4 concludes. All proofs are presented in

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the responsibility for climatic change of developed countries between 53%-61%, and for developing countries approximately 39%-47%. Similarly, according to Zhang et al (2008), between 1850 and 2004 G8 countries accounted for 61% of GHG emissions.

the appendix.

2 The model

We consider two different regions, 1 and 2, that determine the flow of current emissions, $E^i(\tau)$, which is a necessary input in productive activities. We ignore any other input or technology accumulation process, and hence the flow of benefits from production is fully determined by current emissions $w^i(E^i(\tau))$, with $(w^i)'(E^i(\tau)) > 0$ and $(w^i)''(E^i(\tau)) < 0$. The emissions in both regions give rise to a stock of pollution according to the dynamics equation:[2]

$$\dot{P}(\tau) = E^1(\tau) + E^2(\tau) - \delta P(\tau), \quad P(0) = P_0, \tag{1}$$

with δ the degree of assimilative capacity of the environment, and P_0 the initial pollution stock.

We analyze a cooperative agreement within a finite period $[0, T]$, which would overcome the tragedy of the commons, inducing a reduction in the flow of emissions (and in current benefits), in exchange for greater joint gains of a cleaner environment. The optimal emissions under this cooperative game are given by the solution to the optimization problem:

$$\max_{E^i,\, i \in \{1,2\}} \sum_{i=1}^{2} \left\{ \int_0^T w^i(E^i(\tau)) e^{-\rho\tau} d\tau - D^i(P(T)) e^{-\rho T} \right\}, \tag{2}$$

subject to the stock pollution dynamics in (1). The value that each region assigns to the environment is collected in the scrap value $-D^i(P(T))$. The higher grows the pollution stock within the period $[0, T]$, the stronger is the damage born by region i at time T, $(D^i)'(P(T)) > 0$. Note that we are dealing with a stock pollution problem, and hence, the accumulated emissions within the planning horizon will generate lasting effects from T on. Alternatively, we could have also added an instantaneous damage of pollution while cooperation runs, defining instantaneous payoff as $w^i(E^i(\tau), P(\tau))$. We avoid this as it would complicate the exposition, without adding more interesting insights to the main fact that we

[2] A superscript in a given variable refers to the specific region. Variables without superscript indicate global quantities for the two regions jointly considered.

want to capture: cooperation implies current sacrifices in exchange for a better future environment. And the two regions differ on the damage they born from the environmental problem.

The optimal cooperative emissions and pollution stock at time $t \in [0,T]$ are denoted by $E^i_{\mathrm{C}}(t)$ and $P_{\mathrm{C}}(t)$, respectively. Correspondingly, at any time t the total payoff to go that region i gets if cooperation is maintained from this moment till the end of the planning horizon, T, reads:

$$W^i_{\mathrm{C}}(t) = \int_t^T w^i(E^i_{\mathrm{C}}(\tau))e^{-\rho(\tau-t)}d\tau - D^i(P_{\mathrm{C}}(T))e^{-\rho(T-t)}. \tag{3}$$

At any intermediate time $t \in [0,T]$ we want to find out what each region contributes to the agreement (in the form of foregone emissions/production benefits), and what each region gains from the agreement (in the form of a cleaner environment). To that aim the cooperative solution must be compared against the non-cooperative solution from this moment on. Assuming that cooperation has been maintained up until time t, and the two regions play non-cooperatively from this moment on, each region $i \in \{1,2\}$ solves the maximization problem:

$$\max_{E^i} \int_t^T w^i(E^i(\tau))e^{-\rho(\tau-t)}d\tau - D^i(P(T))e^{-\rho(T-t)}, \tag{4}$$

$$\text{s.t.: } \dot{P} = E^i + E^{-i} - \delta P, \quad P(t) = P_{\mathrm{C}}(t). \tag{5}$$

The feedback Nash equilibrium of this non-cooperative game starting at time t, will be denoted by $E^i_{\mathrm{N}}(\tau;t)$ for $\tau \in [t,T]$. Correspondingly, $P_{\mathrm{N}}(\tau;t)$ represents the optimal pollution stock path under the Nash equilibrium. The total payoff to go that region i gets if it deviates from cooperation at time t and both players play non-cooperatively therein is:[3]

$$W^i_{\mathrm{N}}(t) = \int_t^T w^i_{\mathrm{N}}(\tau;t)e^{-\rho(\tau-t)}d\tau - D^i(P_{\mathrm{N}}(T;t))e^{-\rho(T-t)}, \tag{6}$$

As it is common in the Literature, we assume that once cooperation is halted, the two regions play non-cooperatively henceforth, at least till time T. Alternatively, one could argue that they could, for example, decide to renegotiate an agreement at any time after t and prior to T. Such a possibility is, for example, introduced

[3]We use notation $w^i_{\mathrm{N}}(\tau;t)$ instead of $w^i(E^i_{\mathrm{N}}(\tau;t))$ for conciseness.

by Sorger (2006) who proposed a immediate renegotiation when the agreement is broken.[4]

It is now possible to define how much region i contributes to the cooperative agreement in the case that each region receives its cooperative payoffs, given by $w^i(E^i_{\mathrm{C}}(\tau))$. This region's contribution would be:[5]

$$C^i(t) = \int_t^T \left[w^i_{\mathrm{N}}(\tau;t) - w^i_{\mathrm{C}}(\tau) \right] e^{-\rho(\tau - t)} d\tau, \quad t \in [0, T]$$

Likewise, the accrued benefits from cooperation from time t on for region i is defined as the reduction in the environmental damage from a less polluted environment associated with lower emissions under cooperation, from t to T:

$$B^i(t) = \left[D^i(P_{\mathrm{N}}(T;t)) - D^i(P_{\mathrm{C}}(T)) \right] e^{-\rho(T-t)}, \quad t \in [0, T] \tag{7}$$

In the non-cooperative solution, optimal emissions in one region are computed taking into account how these emissions increase the future pollution stock, hence implying a stronger environmental damage at the end of the planning horizon. By contrast, in the cooperative solution, the negative effect of pollution on the other region's welfare is also taken into account. In consequence, under cooperation emissions are kept lower as well as the pollution stock at any time and interestingly at the final time. Under cooperation, since both regions emit less, they receive lower instantaneous payoffs: $w^i_{\mathrm{N}}(\tau;t) > w^i_{\mathrm{C}}(\tau)$, for all $i \in \{1,2\}$, and $\tau \in [t,T]$. Therefore $C^i(t) > 0$, for all $t \in [0,T]$. By contrast, a lower pollution stock will imply positive gains from the prospect of a future cleaner environment: $B^i(t) > 0$ for all $i \in \{1,2\}$, $t \in [0,T]$. Provided that the benefits from cooperation come at the end of the cooperative period, and are determined by each region's valuation of the state of the environment, they cannot be redistributed. Therefore, only the costs or each region's contribution can be redistributed.

The main objective of the paper is to define an imputation distribution procedure (IDP) of the payoffs associated to current emissions under cooperation, in

[4]And yet, these is not the only option, the new agreement could be renegotiated with a delay, or signed and broken many times within this period, or its length could be modified to a longer or shorter period, etc.

[5]Again we use notation $w^i_{\mathrm{N}}(\tau;t)$ and $w^i_{\mathrm{C}}(\tau)$ instead of $w^i(E^i_{\mathrm{N}}(\tau;t))$ and $w^i(E^i_{\mathrm{C}}(\tau))$ for conciseness.

order to satisfy some desired axioms. This distribution scheme, $\pi^i(\tau)$, must first fulfill a feasibility condition at any time:

$$\sum_{i=1}^{2} \pi^i(\tau) = \sum_{i=1}^{2} w_{\mathrm{C}}^i(\tau), \quad \forall \tau \in [0,T]. \tag{8}$$

At each time $t \in [0,T]]$, the instantaneous the sharing rule, $\pi^i(\tau)$, distributes the total payoff at this time between the two players. Payoffs can not be borrowed from/lent to the future. In the same way as in the cooperative case in (3) or the non-cooperative case in (6), once the sharing scheme is applied, the payoff to go under the IDP can be computed as:

$$W_{\pi}^i(t) = \int_t^T \pi^i(\tau) e^{-\rho(\tau-t)} d\tau - D^i(P_{\mathrm{C}}(T)) e^{-\rho(T-t)}. \tag{9}$$

We add a subscript π to denote that we are referring to the payoff to go once the distribution procedure is implemented. Likewise, the contribution to the cooperative agreement of region i from any time t on must be re-defined as the difference between the accumulated benefits associated with non-cooperative emissions and the payoffs proposed by the IDP from time t on:

$$C_{\pi}^i(t) = \int_t^T \left[w_{\mathrm{N}}^i(\tau;t) - \pi^i(\tau) \right] e^{-\rho(\tau-t)} d\tau, \quad t \in [0,T] \tag{10}$$

Note that under condition (8) the joint contribution for the two players is the same with and without the distribution procedure:

$$C_{\pi}(t) = \sum_{i=1}^{2} C_{\pi}^i(t) = \sum_{i=1}^{2} C^i(t) = C(t).$$

While the distribution scheme determines each region' contribution $C_{\pi}^i(t)$, it has no effect on the accrued benefits obtained at the end of the cooperative period and hence its expression in (7) remains valid for any IDP. Comparing the payoff to go in the non-cooperative solution in (6) with the IDP in (9), and taking into account (7) and (10) the surplus to go for region i reads:

$$S^i(t) = W_{\pi}^i(t) - W_{\mathrm{N}}^i(t) = B^i(t) - C_{\pi}^i(t), \quad \forall i \in \{1,2\},\ t \in [0,T).$$

For the two countries to sign an agreement we assume that the agreement satisfies global rationality, i.e. $\sum_{i=1}^{2} S^i(0) > 0$. Moreover, we assume that the agreement is jointly profitable initially and at any intermediate time:

Assumption 1 *The global surplus to go linked to the cooperative solution is positive initially and at any ulterior time, that is for all $t \in [0,T)$:*[6]

$$S(t) = \sum_{i=1}^{2}\left(W_C^i(t) - W_N^i(t)\right) = \sum_{i=1}^{2}\left(W_\pi^i(t) - W_N^i(t)\right) = \sum_{i=1}^{2}\left(B^i(t) - C_\pi^i(t)\right) = B(t) - C_\pi(t) > 0.$$

Together with the feasibility condition (8) and the assumption of global rationality without which the agreement would have not been signed, we require the IDP to also satisfy the next three desired axioms.

Axiom 1 (Time consistency) *At any intermediate time t, and for each region i, the payoff to go under the distribution scheme is not lower than the payoff to go in the non-cooperative scenario:*

$$W_\pi^i(t) \geq W_N^i(t), \quad \forall t \in [0,T],\ \forall i \in \{1,2\}.$$

Axiom 2 (Benefits pay principle-BPP) *Positive correlation between the relative gains that each region gets from cooperation and its relative contribution. For any $t \geq 0$,*

$$\left.\frac{\partial \hat{C}_\pi^i(t)}{\partial \hat{B}^i(t)}\right|_{B(t)=Cte} > 0, \tag{11}$$

where

$$\hat{C}_\pi^i(t) = \frac{C_\pi^i(t)}{C(t)}, \quad \hat{B}^i(t) = \frac{B^i(t)}{B(t)}.$$

a hat denoting the relative value for one region with respect to the total.

Moreover, we would like the distribution scheme to also take into account the each region's responsibility from past emissions (which will be defined in Subsection 2.2).

Axiom 3 (Responsibility or poluter's pay principal-PPP) *Defining R^i as region $i-th$'s responsibility for the damage caused by past emissions, the greater its responsibility, the greater must be its relative contribution.*

$$\frac{\partial \hat{C}_\pi^i(t)}{\partial R^i} > 0. \tag{12}$$

[6]Under the feasibility condition (8), and given the definitions in (6) and (9), it immediately follows that $W_\pi(t) = \sum_i W_\pi^i(t) = \sum_i W_C^i(t) = W_C(t)$, $\forall t \in [0,T]$.

2.1 A time-consistent IDP

Assumption 1 establishes overall rationality or Kaldor-Hicks efficiency. Globally, for the two regions, the payoff to go maintaining cooperation surpasses the payoff to go in the non-cooperative scenario. Furthermore, we need to define a distribution scheme that satisfies time consistency, that is, individual rationality in every subgame along the cooperative trajectory.

To characterize this distribution scheme, we define an imputation distribution procedure (IDP), as a flow of payoffs, $\pi^i(\tau)$, for any region $i \in \{1,2\}$ and at any time $\tau \in [0,T]$. This IDP must first satisfy condition (8), according to which the instantaneous joint payoff for the two regions under the IDP equates their cooperative joint payoff. Thus, the IDP must determine how to share the joint total cooperative payoff at each time τ. Furthermore, this sharing rule must also guarantee that, at every time t, both players prefer to follow the cooperative behavior rather than the non-cooperative one as described in next definition.

Definition 4 *An IDP, $\pi^i(\tau)$, with corresponding payoff to go, $W^i_\pi(t)$ in (9), would be time consistent if the following condition is satisfied at any time $t \in [0,T]$:*[7]

$$W^i_\pi(t) = W^i_N(t) + \phi^i(t)S(t) \qquad \forall t \in [0,T], \tag{13}$$

with $\phi^i(t)$ a differentiable function satisfying:

$$\phi^i(t) \in [0,1], \quad \phi^i(t) + \phi^{-i}(t) = 1, \quad \forall t \in [0,T],\ i \in \{1,2\}. \tag{14}$$

Under Assumption 1 of a positive surplus to go t, $W^i_\pi(t) \geq W^i_{\mathrm{N}}(t)$ for any time $t \in [0,T]$, for any region $i \in \{1,2\}$, and for any function $\phi^i(t)$ satisfying (14). Therefore, the time-consistent IDP described in this definition is not unique. Moreover, the joint payoff to go under the IDP equates the cooperative payoff to go at any time t: $W_\pi(t) = \sum_i W^i_\pi(t) = \sum_i W^i_{\mathrm{C}}(t) = W_{\mathrm{C}}(t)$, $\forall t \in [0,T]$. Finally, computing the derivative with respect to t in both sides of this equation one gets that condition (8) holds at every time.

[7]Again, subscript π refers to the cooperative payoff if the IDP is implemented.

Remark 5 *An alternative way to write condition (13) is*

$$\phi^i(t) = \frac{W^i_\pi(t) - W^i_N(t)}{S(t)} = \frac{B^i(t) - C^i_\pi(t)}{B(t) - C(t)}, \quad \forall t \in [0,T], \ \forall i \in \{d,r\}. \tag{15}$$

From this expression, $\phi^i(t)$ *can be interpreted as a measure of the net gains for player* i *in relative terms to the total surplus to go, if cooperation is maintained with under IDP* $\pi^i(t)$ *from any time* $t \in [0,T]$ *on.*

According to previous remark, choosing a differentiable function $\phi^i(t)$, $t \in [0,T]$, satisfying conditions in (14) fully determines the net gains that each region gets from the agreement. Therefore, given this function we can univocally characterize the flow of current benefits under the IDP, $\pi^i(t)$.

Proposition 6 *Consider* $\phi^i(t)$ *a differentiable function satisfying (14), the expression of* $W^i_\pi(t)$ *for player* i *at any time* $t \in [0,T)$ *satisfies (8), (9) and (13) under the IDP given by:*[8]

$$\pi^i(t) = w^i_C(t) + \phi^i(t) IVC(t) - IVC^i(t) - \dot{\phi}^i(t) S(t), \tag{16}$$

with $IVC^i(t)$*, the instantaneous value of cooperation at time* t*, for region* i *defined as:*

$$IVC^i(t) = w^i_C(t) - w^i_N(t) + \int_t^T \dot{w}^i_N(\tau;t) e^{-\rho(\tau-t)} d\tau + \left(SV^i\right)' \left(P_N(T;t)\right) \dot{P}_N(T;t) e^{-\rho(T-t)}. \tag{17}$$

and $IVC(t) = \sum_{i=1}^2 IVC^i(t)$.

Proof. See Appendix. ■

At time t, if cooperation has been maintained till then, and if cooperation does not halt at this time, region i would get a typically lower instantaneous payoff linked to lower emissions: $w^i_{\mathrm{C}}(t) - w^i_{\mathrm{N}}(t) < 0$ (instantaneous cost of cooperation at time t). However, because cooperative regions emit less at this time t, this would allow for higher future emissions for country i if the deviation from cooperation takes place at the instant immediately afterwards (the integral term in expression

[8] The term $w^i_{\mathrm{N}}(t)$ refers to the non-cooperative instantaneous payoff when cooperation just halted at exactly time t, i.e. $w^i_{\mathrm{N}}(t;t)$

(17)). Finally, the lower instantaneous emissions at time t also induce a lower pollution stock at time T and therefore a lower environmental damage born by this region from T on (last term in expression (17)).

According to that interpretation of $IVC^i(t)$, the IDP in (16) grants the $i-$th player his instantaneous cooperative payoff, plus the gap between the share $\phi^i(t)$ of the joint instantaneous value of cooperation and the $i-$th player's instantaneous value of cooperation. Thus, if the $i-$th IVC is larger than its share of the total IVC, this region would transfer part of its instantaneous payoff to its opponent. Further, the $i-$th instantaneous payoff is reduced in the speed at which his share of the total surplus to go increases.

Moreover, from (16) one gets that the feasibility condition (8), is also guaranteed, which implies that the instantaneous payoff that the IDP distributes between the two players matches the total instantaneous cooperative payoff at every time t. Therefore, the instantaneous side payment to region i can be computed as:

$$sd^i(t) = \pi^i(t) - w_{\mathrm{C}}^i(t) = \phi^i(t)IVC(t) - IVC^i(t) - \dot{\phi}^i(t)S(t).$$

It is obviously true that $sd^i(t) + sd^{-i}(t) = 0$. Thus, $sd^i(t) > 0$ defines an instantaneous transfer to region i, while $sd^i(t) < 0$ represents a transfer from region i to the region $-i$. Similarly, we can define the total transfer to region i from a give time t on as:

$$SD^i(t) = \int_t^T sd^i(\tau)e^{-\rho(t-T)}d\tau.$$

Any differentiable function $\phi^i(t)$, satisfying $\phi^i(t) = 1 - \phi^{-i}(t)$ and $\phi^i(t) \in [0,1]$ for all $i \in \{1,2\}$ and for all $t \in [0,T]$, guarantees time-consistency. Among these functions, we look for the specification(s) which also implies the satisfaction of axioms 2 and 3.

2.2 An IDP satisfying BPP and PPP: axioms 2 and 3

If we ignore the PPP or responsibility axiom and focus only on the BPP, we observe that by defining $\phi^i(t)$ equal to $\hat{B}^i(t)$, relative contributions and relative benefits are identical, immediately implying Axiom 2.

Lemma 7 *The assumption* $\phi^i(t) = \hat{B}^i(t)$ *implies* $\hat{C}^i_\pi(t) = \hat{B}^i(t)$ *and, hence, an IDP that straightforwardly satisfies the benefits pay principle stated in axiom 2.*

Proof. Assuming $\phi^i(t) = \hat{B}^i(t)$ and plugging this into (15) one can easily conclude the statement. ■

A distribution procedure which required an exact equivalence between relative gains and relative contributions would be uniquely characterized by function $\phi^i(t) = \hat{B}^i(t)$. However, we seek for an IDP which satisfies positive correlation between relative gains and relative contributions, while at the same time, these relative contributions also take into account the region's responsibility from past emissions.

Let's define region i's net responsibility as the damage that this region's past emissions has caused to region $-i$, minus the damage that region $-i$'s accumulated past emissions has caused to region i. This definition of responsibility takes into account three mayor ingredients highlighted, for example, in Hayner and Weisbach (2016): who and to what extent causes the problem; which is the size of the harm caused; and to what extent each region has been impacted. Responsibility can be equivalently defined as the total damaged caused by region i which is not born by this region. Define by r^i all past emissions from region i divided by all past emissions, i.e. the percentage of the initial pollution stock country i is responsible for. Defining the damage born by region i from all previous emissions at the damage associated with the initial pollution stock, $D^i(P_0)$, this region's responsibility would read:

$$R^i = r^i \hat{D}^{-i}(P_0) - r^{-i}\hat{D}^i(P_0), \quad \text{with} \quad \hat{D}^i(P_0) = \frac{D^i(P_0)}{\sum_{i=1}^2 D^i(P_0)}. \tag{18}$$

Or alternatively,

$$R^i = \frac{r^i \sum_{i=1}^2 D^i(P_0) - D^i(P_0)}{\sum_{i=1}^2 D^i(P_0)} = r^i - \widehat{D}^i(P_0). \tag{19}$$

The two expression (18) and (19) are equivalent and it is immediately obvious that $R^i = -R^{-i}$. In general the two regions are both responsible from past emissions although probably at different scales. However, we will say that region i is responsible (or more responsible) if $R^i > 0$ and not responsible (or less responsible if $R^i < 0$).

Given this definition of responsibility we propose a linear specification for $\phi^i(t)$ as a function of $\hat{B}^i(t)$ and R^i:

$$\phi^i(t) = \hat{B}^i(t) + \alpha R^{-i} = \hat{B}^i(t) - \alpha R^i. \tag{20}$$

Lemma 8 *To guarantee that $\phi^i(t) \in [0,1]$ one needs to impose constraints on the values of α.*

$$0 \leq \alpha \leq \max\left\{\frac{\hat{B}^1}{R^1}, \frac{\hat{B}^2}{R^2}\right\} \equiv \alpha_{\max}.$$

Proof. From the definition of $\phi^i(t)$ in (20) it always holds that $\phi^i(t) + \phi^{-i}(t) = 1$. Assuming $\alpha > 0$, if $R^i > 0$, then $\phi^i(t) > 0$ requires $\alpha < \hat{B}^i/R^i$, while $\phi^i(t) < 1$ always holds. If, conversely, $R^i < 0$, then $\phi^i(t) < 1$ requires $\alpha < \hat{B}^{-i}/R^{-i}$, while $\phi^i(t) > 1$ always holds. Then the upper bound for α follows. ■

Plugging this new definition of $\phi^i(t)$ into (15), the contribution of region i in relative and in absolute terms read:

$$\hat{C}^i_\pi(t) = \hat{B}^i(t) + \alpha\frac{S(t)}{C(t)}R^i, \tag{21}$$

$$C^i_\pi(t) = B^i(t)\frac{C(t)}{B(t)} + \alpha R^i S(t), \quad \forall \alpha \in [0, \alpha_{\max}]. \tag{22}$$

The relative contribution of region i in (21) is determined by its relative gains from cooperation increased or decreased depending on whether the region is or is not responsible from past emissions. In absolute terms, expression (22) shows that at time t, each region contributes an equal share, $C(t)/B(t)$ of its benefit from cooperation plus the share $\alpha|R^i|$ of the the surplus to go if the region is responsible (i.e. $R^i > 0$); or minus this share if it is not responsible (i.e. $R^i < 0$).

Proposition 9 *The IDP $\pi^i(t)$, defined in Proposition 6, with the proposed specification for $\phi^i(t)$ in (20) is time consistent and satisfies the benefits pay principle and the polluters pay principle: Axioms 1-3.*

Proof. Time consistency is proved in Proposition 6. The proof of Axioms 2 follows by noting that the term $\alpha S(t)/C(t)$ in (21) is unaltered if $\hat{B}^i$ increases and $\hat{B}^{-i}$ decreases keeping $B(t)$ unchanged. Likewise, the proof of Axioms 3 is straightforward by noting that the term $\alpha S(t)/C(t)$ in (21) does not depend on R^i. Therefore,

$$\left.\frac{\partial \hat{C}^i_\pi(t)}{\partial \hat{B}^i(t)}\right|_{B(t)=Cte} = 1 > 0, \qquad \frac{\partial \hat{C}^i_\pi(t)}{\partial R^i} = \alpha\frac{S(t)}{C(t)} > 0.$$

And Axioms 2 and 3 follow. ■

Remark 10 *From (21) it is straightforward to conclude that:*

1. *If* $\alpha = 0$*, or if* $R^i = R^{-i} = 0$ *(i.e.* $r^i \hat{D}^{-i}(P_0) = r^{-i}\hat{D}^i(P_0)$*), then:*

$$\hat{C}_\pi^i(t) = \hat{B}^i(t), \qquad C_\pi^i(t) = B^i(t)\frac{C(t)}{B(t)}.$$

2. *If* $\alpha = \alpha_{\max}$*, and* $R^i > 0$*, then:*

$$C_\pi^i(t) = B^i(t), \quad C^{-i}(t) = B^{-i}(t) - S(t).$$

If all the weight is given to the benefits pay principle ($\alpha = 0$), or both regions are equally responsible ($R^i = 0$), then the relative contribution matches the relative benefit from cooperation for both regions. Each region pays the same percentage of its benefits from cooperation.

If the IDP gives the maximum possible weight to responsibility, $\alpha_{\max}$, and if region i is responsible, the contribution of this region from any given time t on equals its benefits from cooperation from this moment on. This is the maximum amount that this region can contribute compatible with time consistency. On the other hand, the contribution of region $-i$ who is not responsible, is given by its benefits decreased by the total surplus from this time on.

Proposition 11 *For the proposed IDP in (16), (17) and (20), the egalitarian rule,* $\phi^i(t) = 1/2$ *for all* $t \in [0, T)$ *does not satisfy axioms 1, 2 and 3.*

Corollary 12 *Indeed, the egalitarian rule does not satisfy axioms 1, 2 and 3 for any* $\phi(\hat{B}^i, R^i)$ *satisfying* $\partial\phi/\partial\hat{B}^i > 0$*.*

3 Numerical example

In this section, we describe how the IDP presented in the previous section can be applied, considering an example in which region 1 is more responsible for the current environmental problem and is less hardly hit by it than region 2.

An example could be the global warming problem.[9] Analyzing what are the production payoffs associated with current emissions, the future losses from a 2 or 3 degree rise in temperature, or the responsibility from past emissions, distinguishing for different countries is an extraordinarily challenging task we do not undertake. Instead, we illustrate the proposed IDP for a toy model using some of the imperfect figures offered in the literature. We divide the world in two regions, one encompassing industrialized countries and economies in transition or annex I countries, and the other non-annex I countries. Assume that region 1 corresponds to annex I countries, which are more responsible from past emissions $R^1 > 0$, while $R^2 = -R^1 < 0$. Correspondingly, region 2 which encompasses non-annex I countries seems to be facing a stronger risk from global warming. Their losses from the rise in temperatures associated with global warming would be higher than losses in region 1. Consequently, region 2 has more to gain if the agreement is signed and maintained: $\hat{D}^1(P) < \hat{D}^2(P)$, for any $P > 0$.

For simplicity, we assume that the instantaneous benefits linked to current emissions are described by a linear quadratic function:

$$w(E^i(\tau)) = a^i E^i(\tau) - \frac{(E^i)^2(\tau)}{2}.$$

We make the assumption that $a^2 = a^1 = a$ to clearly state that the two countries only differ in their responsibility from past emissions and their valuation of a cleaner environment. Thus, the marginal gains for additional emissions or the cost of abatement are identical in both regions.[10] The scrap value is considered linear in the stock of pollution at time T: $D^i(P(T)) = d^i P(T)$, with $d^2 \geq d^1 > 0$, showing that region 2 will be more strongly hit by the environmental problem.

The simulation is run considering the following parameter values:

$$r^1 = .72,\ (r^2 = .28),\ d^1 = .5 < d^2 = .6,\ a = 2,\ \delta = .1,\ P_0 = 10,\ \rho = .03.$$

We are considering that the ratio of relative past emissions for developed countries

[9] More generally, consider two regions who share a polluted environment. Region 1's responsibility exceeds its share of the burden from current pollution: $r^1 - \hat{D}^i(P_0) > 0$ and hence, $R^1 = -R^2 > 0$. Moreover, region 2 is more hardly hit by the problem than region 1, or equivalenty it will bear a higher damage if no agreement on emissions reduction is implemented.

[10] This assumption can be easily removed, introducing asymmetry, to study the effect of a rise in a^i.

in region 1 is given by $r^1 = 0.72$, (correspondingly $r^2 = 0.28$). For this simplified model, we have a series of results.

Results:

- From the simplifying assumption of a linear damage, the non-cooperative game is linear state and the open-loop and feedback solutions coincide. This greatly simplifies the resolution and the comparison of results.

- From the simplifying assumption of a linear damage it also follows that the relative damage from pollution does not depend on the level reached by the stock of pollution: $\hat{D}^i(P) = d^i/(\sum_{j=1}^2 d^j)$. In consequence, the net responsibility of region i reads:

$$R^i = r^i - \frac{d^i}{\sum_{j=1}^2 d^j}.$$

  Region 1 with $r^1 > 1/2$ and $d^i/(\sum_{j=1}^2 d^j) < 1/2$, has a positive net responsibility, $R^1 > 0$, while conversely, for region 2, $R^2 = -R^1 < 0$. In particular, $R^1 = 0.265$, $R^2 = -0.265$.

- Given the definition of $\phi^i(t)$ in (20), and the assumption of a linear damage function it easily follows that:

$$\phi^i(t) = \frac{D^i(P_{\mathrm{N}}(T;t)) - D^i(P_{\mathrm{C}}(T))}{\sum_{j=1}^2 (D^j(P_{\mathrm{N}}(T;t)) - D^j(P_{\mathrm{C}}(T)))} - \alpha\left(r^i - \hat{D}^i(P_0)\right) = \frac{d^i}{\sum_{j=1}^2 d^j} - \alpha\left(r^i - \frac{d^i}{\sum_{j=1}^2 d^j}\right).$$

  Because the relative damage under the linear hypothesis is independent of the level reached by the stock of pollution, $\phi^i(t)$ is the same constant at any time $t \in [0, T]$. For region 1, $r^1 = 0.72 > 1/2$ and $d^i/(\sum_{j=1}^2 d^j) = 0.45 < 1/2$, and the last term in brackets is positive. In consequence, $\phi^1(t) < d^i/(\sum_{j=1}^2 d^j) = 0.45$. As shown in Figure 1, the greater the weight given to responsibility, α, the greater the share of the total surplus to go for region 1 and the lower the share to region 2. In either case, regardless of the value of α, region 1 never get 1/2 of the surplus to go and region 2 always gets more than 1/2 of the surplus. Thus, the egalitarian rule $\phi^i = 1/2$ cannot arise under this setting, provided that one region is more responsible from and, at the same time, less strongly hit by the environmental problem.

- The agreement is time consistent as long as $\phi^i(t)$ remains positive for any $i \in \{1, 2\}$ and any $t \in [0, T]$. To that aim, the value of α needs to be lower

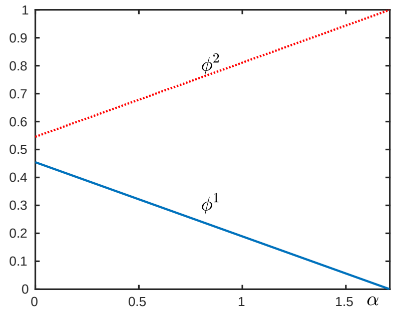

Figure 1: $\phi(t)$ for different α.

than $\alpha_{\max}$. Since region 1 is the region more responsible from past emissions (i.e. $R^1 > 0$), then the upper bound for α is defined by:

$$\alpha_{\max} = \frac{\hat{B}^1}{R^1} = \frac{d^1}{r^1 d^2 - r^2 d^1} = 1.7.$$

- Figure 2 shows that as the weight, α, given to responsibility increases, also the contribution of the region more responsible from past emissions rises in relative terms to the contributions of the less responsible region. There exists a value, $\tilde{\alpha}$, at which the two regions contribute the same:

$$C_\pi^1(t) = C_\pi^2(t), \quad \alpha = \tilde{\alpha} = \frac{(d^2 - d^1)((d^1)^2 + (d^2)^2 + 3d^1 d^2)}{R^1((d^1)^2 + (d^2)^2)} = 0.51 > 0. \quad (23)$$

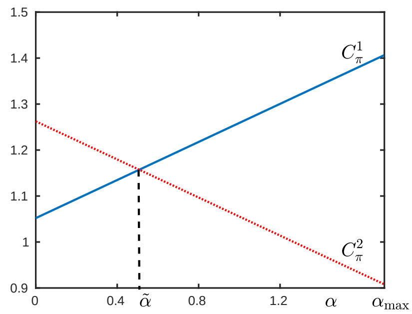

Figure 2: $C_\pi^i(t)$ for different α.

- As the weight given to responsibility increases, region 1 receives a lower total transfer from region 2 or start paying a transfer to this region. This is depicted in Figure 3, where $SD^2(t)$ represents the total transfer that region 2 receives from (or pays to if negative) region 1 from time t till the end of the cooperative period T. For small values of α, responsibility is not much considered and the prevailing force is the BPP. Therefore, region 2 which gains more from cooperation pays a transfer to region 1. By contrast, for large values of α the PPP becomes preponderant and it is the region more responsible from past emissions, region 1, which pays a transfer to region 2. There exists a value $\hat{\alpha}$ at which the two principles balance and no transfer in one direction or the other is needed. This value can be computed in our example as:

$$SD^1(t) = SD^2(t) = 0, \quad \alpha = \hat{\alpha} = \frac{(d^2 - d^1)((d^1)^2 + (d^2)^2 + 4d^1d^2)}{2R^1(d^1 + d^2)((d^1)^2 + (d^2)^2)} = 0.85 > 0. \tag{24}$$

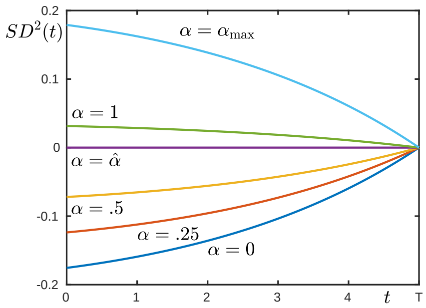

Figure 3: $SD^1(t)$ for different α.

Comparing expressions in (23) and (24) the gap $\hat{\alpha} - \tilde{\alpha}$ can be written as

$$\hat{\alpha} - \tilde{\alpha} = \frac{(d^2)^2 - (d^1)^2}{2R^1((d^1)^2 + (d^2)^2)} = 0.34 > 0.$$

$\hat{\alpha} > \tilde{\alpha}$ implies that a solution with an equal contribution by the two regions requires a transfer from region 2 to region 1. Or equivalently, a solution with no transfer would be associated with a higher contribution by the more responsible region 1.

- The effect of α on instantaneous payoffs can be observed in Figure 4. The instantaneous payoff is the greatest with no cooperation (the dash-dotted red line), when the two regions do not make any effort in emissions reductions. Cooperation comes with the associated cost of lower emissions throughout the whole cooperative period and the subsequent contribution of each region. Cooperation will be profitable because it will imply a strong reduction in environmental damage from T on which, in aggregate terms for the two regions, overcomes the aggregate contribution, i.e. $S(t) > 0$. The cooperative solution, without any side-payment is depicted by the solid black line in Figure 4. An IDP that disregarded responsibility, $\alpha = 0$, would imply lower effort to region 1, which gets higher payoffs than without any redistribution scheme, and a higher effort to region 2, which gets lower payoffs. This region transfer part or its cooperative payoff to region 1. By contrast, when the responsibility principle is considered at its maximum, $\alpha = \alpha_{\max}$, then the situation is reversed. Region 1 has to increase its effort while region 2 reduces its contribution. In the two cases and any intermediate α, the agreement is time consistent, and satisfies the two axioms of BPP and PPP. In this particular example, the agreement would also be time consistent for $\alpha = \hat{\alpha}$ at which no transfer takes place and each region receives its cooperative payoff without any side-payment.

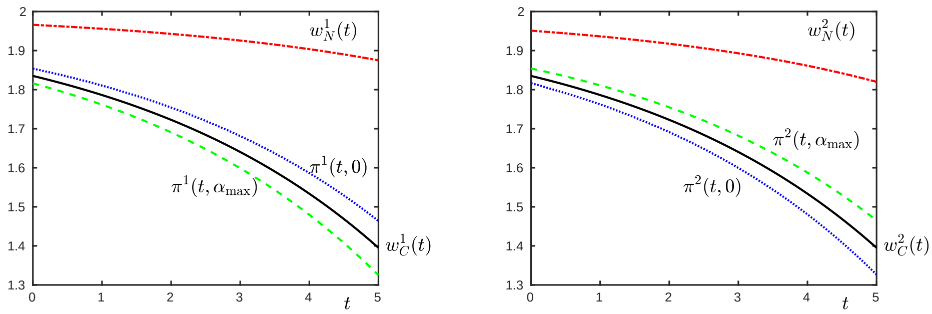

Figure 4: $w^i_{\mathrm{C}}(t)$, $w^i_{\mathrm{N}}(t)$, $\pi^i(t), (\alpha = 0)$, $\pi^i(t), (\alpha = \alpha_{\max})$.

4 Conclusions

We propose an imputation distribution procedure which defines how to share the efforts, in the form of emissions reductions, in an temporal environmental agreement. The IDP is of interest when the two parts differently value the environment, i.e. are differently benefited by the agreement. Additionally, the sharing rule also takes into account each regions's responsibility from past emissions.

The proposed sharing mechanism guarantees the time-consistency of the cooperative solution, since the cooperative parts prefer to remain in the agreement at any intermediate time, rather than to deviate to a non-cooperative mode of play from this time on. Moreover, our proposed mechanism also satisfies two desirable properties. First, it satisfy a benefits pay principle: the more one region benefits from the agreement, the higher is its relative contribution. Finally, it also satisfy a polluters pay principle: the more responsible a region is from past emissions, the greater is its relative contribution.

We do not base our sharing scheme on a specific cooperative solution concept, but rather present a general formulation and require the IDP to satisfy time-consistecy, the BPP and the PPP. As a result, we obtain a family of sharing rules satisfying all these properties. The relative weight that shall be given to the two principles is still an open question.

Appendix

Proof proposition 6

Computing the time derivatives in (9) and (13) we get:

$$\dot{W}^i_\pi = -\pi^i + \rho W^i_\pi, \quad \dot{W}^i_\pi = \dot{W}^i_N + \dot{\phi}^i S + \phi^i \dot{S}.$$

And computing the time derivatives in (3) and (6)

$$\dot{W}^i_{\mathrm{C}} = -w^i_{\mathrm{C}} + \rho W^i_{\mathrm{C}},$$

$$\dot{W}^i_{\mathrm{N}} = -w^i_{\mathrm{N}} + \rho W^i_{\mathrm{N}} + \int_t^T \dot{w}^i_{\mathrm{N}}(\tau;t) e^{-\rho(\tau-t)} d\tau + \left(SV^i\right)' \left(P_{\mathrm{N}}(T;t)\right) \dot{P}_{\mathrm{N}}(T;t) e^{-\rho(T-t)}.$$

We call

$$I^i_{\mathrm{N}}(t) = \int_t^T \dot{w}^i_{\mathrm{N}}(\tau;t) e^{-\rho(\tau-t)} d\tau + \left(SV^i\right)' \left(P_{\mathrm{N}}(T;t)\right) \dot{P}_{\mathrm{N}}(T;t) e^{-\rho(T-t)}.$$

Using these equations we get

$$-\pi^i + \rho W^i_\pi = -w^i_{\mathrm{N}} + \rho W^i_{\mathrm{N}} + I^i_{\mathrm{N}} + \dot{\phi}^i S + \phi^i \dot{S} = -w^i_{\mathrm{N}} + \rho(W^i_\pi - \phi^i S) + I^i_{\mathrm{N}} + \dot{\phi}^i S + \phi^i \dot{S},$$

then using that $\dot{S} = \dot{W}^i_{\mathrm{C}} + \dot{W}^{-i}_{\mathrm{C}} - \dot{W}^i_{\mathrm{N}} - \dot{W}^{-i}_{\mathrm{N}}$,

$$\pi^i = w^i_{\mathrm{N}} + \rho\phi^i S - I^i_{\mathrm{N}} - \dot{\phi}^i S - \phi^i \dot{S} - \phi_i[\rho S + w^i_{\mathrm{N}} - w^i_{\mathrm{C}} - I^i_{\mathrm{N}} + w^{-i}_{\mathrm{N}} - w^{-i}_{\mathrm{C}} - I^{-i}_{\mathrm{N}}]$$

Calling

$$IVC^i(t) = w^i_{\mathrm{C}}(t) - w^i_{\mathrm{N}}(t) + \int_t^T \dot{w}^i_{\mathrm{N}}(\tau;t) e^{-\rho(\tau-t)} d\tau + \left(SV^i\right)' \left(P_{\mathrm{N}}(T;t)\right) \dot{P}_{\mathrm{N}}(T;t) e^{-\rho(T-t)},$$

and $IVC(t) = \sum_{i=1}^{2} IVC^i(t)$, we obtain the result. Moreover:

$$\pi^i + \pi^{-i} = w_{\mathrm{C}}^i + w_{\mathrm{C}}^{-i} - IVC - (\dot{\phi}^i + \dot{\phi}^{-i})S + (\phi^i + \phi^{-i})IVC = w_{\mathrm{C}}^i + w_{\mathrm{C}}^{-i}.$$

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