Optimal Government Scrappage Subsidies in the Presence of Strategic Consumers*

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Abstract

Many countries have introduced vehicle scrappage programs to motivate consumers to replace their old cars earlier. Since these programs are offered during a given period of time, policy makers need to plan for inter-temporal subsidies. Considering a two-period game between strategic consumers and the government, we determine the optimal scrappage subsidy levels. Our results demonstrate that the subsidy level in the second period is higher than the first period's, allowing the government to discriminate on price (or subsidy) between consumers with different valuations. In addition, we show that subsidy levels increase with the government's targeted replacement level. However, when the government target level changes from intermediate to high levels, the first-period subsidy drops while the second-period subsidy remains unchanged.

Keywords: Vehicle scrappage program; Strategic consumers; Stackelberg game; Government subsidies.

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1 Introduction

It is a common practice for governments to design incentive programs to accelerate the adoption of new green (or less-polluting), durable products or technologies. For instance, the US government gives consumers who buy a new electric vehicle $2,500 to $7,500 in tax credits (Shao et al., 2017), and the California Solar Incentive (CSI) program invested $2.167 billion on installing solar panels (Cohen et al., 2016). Other programs offer consumers a subsidy for replacing their aging appliances. In Canada, Heating, Ventilation, and Air Conditioning (HVAC) rebates offset the cost of upgrading to a new heating and cooling system. These financial incentives motivate households to replace their old, high-energy-consuming appliance with a more energy-efficient one. One of the most prevalent replacement incentive policies is the vehicle scrappage program, variations of which have been implemented in many countries. In the US, the Cash for Clunkers program provided up to $4,500 to consumers who replaced their old car with a new, more fuel-efficient one. Similar subsidy programs have also been adopted in Canada, Asia, and the European Union (Huang et al., 2014).

While these subsidy programs have both environmental and financial benefits for consumers and suppliers, they are also costly. Therefore, governments should aim at designing subsidy programs that achieve their objectives in the most cost-efficient way. When an incentive program lasts over time, the decision-maker must carefully plan the subsidy schedule. Indeed, the impact of having a constant subsidy versus a varying (decreasing or increasing) subsidy over time is not neutral. For instance, if the subsidy is decreasing over time, then some consumers may accelerate their vehicle replacement, which would most likely result in a smaller environmental gain than if these consumers had waited a while before replacing. In practice, both decreasing and increasing trends are observed in different subsidy programs: while the US federal government subsidy for renewable energy was increasing over time, a vehicle scrappage program in Denmark had a decreasing trend (Jorgensen and Zaccour, 1999).

In this paper, we investigate the problem of designing a subsidy program over time. To keep our model as parsimonious as possible, while still being able to shed an interesting light on subsidy schedules, we retain a two-period game model. The game is played à la Stackelberg, where the government (leader) announces the subsidy program, and consumers (followers) make their replacement decisions. Consumers are assumed to be strategic, that is, they decide whether to keep or replace their car based on their current and future utilities. The problem to solve in our context is more complicated than the one arising a typical purchasing decision by strategic consumers. Indeed, when replacing an old car with a new one, the consumer takes into account the value of both cars, not only the value of the new one. Moreover, a strategic consumer purchasing a new product accounts for the perceived value of the product at different periods. This implies that, in a replacement problem, consumers are not only heterogeneous in terms of their perceived valuation of the new product but also in the age of their currently owned cars. This led us to including two heterogeneities in our model: in consumer valuation and in used car age (quality of the car). Another difference with the typical timing of purchasing decisions is the fact that scrappage programs usually include an eligibility age. This distinguishes vehicle scrappage policies from trade-in programs, where the consumer can trade in a product for a new one, without any age constraint. Hence, it is necessary to differentiate between consumers who are eligible at a given period and those who can get the subsidy only in future periods.

Governments often assign an adoption target to their subsidy programs. For example, in 2011, President Obama announced in 2011 a target of "one million electric vehicles on the road by 2015". Another example is the "1,940 MW of new solar generation capacity" target assigned by the California Solar Incentive program (Cohen et al., 2016). Along these lines, we suppose that the government's objective is to minimize the total cost of the program, subject to satisfying a preset target. Clearly, one can imagine other objectives such as minimizing greenhouse gas emissions or maximizing social welfare. However, Cohen et al. (2016) argue that these optimization problems are equivalent and yield the same results. In this paper, we are also interested in finding out how the target level affects the government's optimal policy.

1.1 Brief literature background

Vehicle scrappage subsidy programs, which were designed to protect the environment and boost the car market, have been the topic of numerous studies over the past two decades. Some have investigated the problem as a static model. In a static framework, Hahn (1995) determines the number of scrapped cars as a function of the subsidy level. In a similar setting, Dill (2004) illustrates that scrapped vehicles are usually driven less than other of the same age. Lavee et al. (2014) analyze the effectiveness of the scrappage program by estimating a supply curve of retired vehicles as a function of the subsidy level. Li and Linn (2013) calculate the number of new vehicle sales resulting from the Cash-for-Clunkers program. They show that 45% of the spending went to consumers who would have replaced their car even without the subsidy.

Unlike the above-cited contributions, some papers have developed dynamic models to design subsidy programs. In the context of the adoption of a new technology with learning-by-doing, Jorgensen and Zaccour (1999) and Janssens and Zaccour (2014) determine the equilibrium path of consumer subsidies in dynamic games involving a government and a firm. Jorgensen and Zaccour (1999) also take into account guaranteed buys by the government, while assuming linear cost learning. Janssens and Zaccour (2014) retain hyperbolic cost learning and compute the total subsidy budget needed to reach a desirable price by the end of the subsidy program. Lobel and Perakis (2011) characterize the trajectory of consumer subsidies for photovoltaic technology.In these papers, dynamic subsidies are offered for purchasing new technologies (or products), but scrappage programs provide incentives for replacing old products with new ones. In this sense, scrappage programs are similar to trade-in subsidies. Zhu et al. (2017) consider a two-period model where a firm makes new products in the first period and collects used products in the second period through a trade-in program. Using a similar setting, Miao et al. (2017) develop three supply chain structures with trade-ins. They also characterize conditions under which trade-ins can promote the environment. Zhang and Zhang (2018) address how consumer purchase behavior affects the economic and environmental benefits of trade-in programs. Likewise, we consider a two-period model where strategic consumers decide whether to trade in their car. However, our paper differs from these contributions by accounting for an eligibility age in the scrappage programs.

Strategic consumer behavior has been extensively studied in the literature. For a comprehensive review of the effect of strategic consumers on pricing, inventory, and information, see Wei and Zhang (2017). A few recent papers have accounted for consumer behavior in subsidy policies. Shiraldi (2011) proposes a dynamic discrete choice model to study automobile replacement decisions by heterogeneous consumers. He argues that some beneficiaries of the scrappage subsidy program would have replaced even without the subsidy. Using a similar setting, Wei and Zhang (2017) show that targeting consumers who would not have replaced their car without a subsidy is the key factor in designing scrappage programs. While in Shiraldi (2011) and Wei and Zhang (2017), the vehicle choices of heterogeneous consumers are studied, Langer and Lemoine (2017) focus on the timing of the decision of strategic consumers. In this regard, Langer and Lemoine (2017) is similar to our paper except that it investigates the effect of the subsidy on new product purchases, rather than on replacing old products as we do here. The closest paper to ours is Zaman and Zaccour (2018), where the effect of consumer heterogeneity on the vehicle replacement decision is addressed. In Zaman and Zaccour (2018), the authors solve the optimization problem of the consumer, and the effects of the subsidies are analyzed ex-post, assuming no strategic role for the government. In this paper, we relax this assumption and let the government act strategically when designing its subsidy program.

1.2 Research questions and contributions

To the best of our knowledge, this paper is the first to study the equilibrium scrappage subsidy policies in the presence of strategic consumers. Our aim is to answer the following questions:

1. How do strategic consumers owning cars of different ages react to current and future subsidies?

2. What are the most cost-efficient policies to reach different replacement target levels?

3. Under what conditions should the government reduce the eligibility age?

We believe that answering these questions can help the government design cost-efficient scrappage programs. Our results are summarized as follows:

1. A consumer's decision to replace depends on both periods' subsidy levels and on the difference between them. When this difference is low, consumers who are eligible in both periods will either replace in the first period or never replace. Apparently, consumers who do not replace at all have a low valuation. Among these consumers, owners of older cars have an even lower valuation than those with younger cars. Note that the quality of a car decreases with age, and those who keep older cars should have a lower valuation than those with younger cars. When the second-period subsidy is large enough compared to the first-period subsidy, some consumers with a low valuation who would not have replaced before, replace in the second period. In this situation, while low-value consumers with younger cars replace in the second period, those with older cars may still prefer to keep their used car. As mentioned above, the older-car owners have a lower valuation than the younger-car owners and may still refuse the subsidies. Therefore, they need more incentive in the second period (i.e., higher second-period subsidy) to opt for replacement in that period rather than no replacement at all.

2. As one might expect, the optimal subsidy policy depends on the car replacement target level. Clearly, a low target can be met with no subsidy. For medium target levels, subsidies in both periods are positive and increasing in the target level, and the second-period subsidy is higher than the first-period one, allowing for price discrimination between high- and low-value consumers. While high-value consumers replace in the first period with a lower subsidy, those with a lower valuation are given higher incentives in the second

period. As the target level increases, there is a threshold at which the first-period subsidy is dropped to a lower value while the second-period subsidy is increased. However, after this threshold, the two subsidy values again increase in the target level. In fact, for a higher target than this threshold, the government needs to induce more low-value consumers to replace in the second period. Consequently, it offsets the higher subsidy cost in the second period by reducing the first-period subsidy level.

3. The government's eligibility age is based on two factors, namely, its target level and the incremental number of cars that become eligible if eligibility age is reduced. Generally, the government is better off to decrease the eligibility age when these two factors are high enough.

2 Model

Consider a two-period model where the government subsidizes consumers who replace their car that are older than a given age by new ones. Denote by p, s_1, and s_2, the price of the new car and the subsidies in the first and second period, respectively. Let v_i be the valuation of consumer i of driving a car. We assume v_i to be independent of the car's quality and age and uniformly distributed between 0 and 1. Denote by $Q_\tau, \tau = 0, \dots \omega$, the quality of a car of age τ, where ω is the maximum age in the vehicle fleet. We suppose, not unrealistically, that $Q_\tau \geq Q_{\tau+1}$, i.e., that car quality decreases with age.

The utility of consumer i owning a car of age τ is postulated to be given by $v_i Q_\tau$. In line with Barahona et al. (2016), this multiplicative form implies that utility is higher for newer cars and for higher-value consumers. Let η be the minimum age to become eligible for the scrappage subsidy program. In our two-period framework, a vehicle can be eligible either in both periods or only in the second one. (There is no point in considering cars that are not eligible in either period, as the program has no effect on their replacement.) Consumers whose vehicles are eligible in both periods can choose between not replacing, replacing in the first period and receiving subsidy s_1, or replacing in the second period and receiving subsidy s_2. Consumers who are not eligible in the first period can benefit from the subsidy program only if they replace their vehicles in the second period.

More specifically, a consumer owning a vehicle aged $\tau \geq \eta$ and replacing it in the first period, enjoys the total discounted utility given by $v_i Q_0 - p + s_1 + \delta v_i Q_0$. If instead she replaces in the second period, then her total discounted utility is $v_i Q_\tau + \delta(v_i Q_0 - p + s_2)$. Never replacing yields the total discounted utility $v_i Q_\tau + \delta v_i Q_{\tau+1}$. Note that, without any loss of qualitative insight, we assume that the quality of a new vehicle remains equal to Q_0 after driving it for one period, i.e., $Q_0 = Q_1$. This assumption, which is not unrealistic, is made to keep the model as parsimonious as possible. The utility of consumers having a vehicle aged $\eta - 1$ is computed in a similar way, while keeping in mind that they are not eligible for the subsidy program in the first period. Table 1 gives the utility in all relevant cases.

	$\tau = \eta - 1$	$\tau \geq \eta$
Replacement in the first period	$v_i Q_0 - p + \delta v_i Q_0$	$v_i Q_0 - p + s_1 + \delta v_i Q_0$
Replacement in the second period	$v_i Q_\tau + \delta(v_i Q_0 - p + s_2)$	$v_i Q_\tau + \delta(v_i Q_0 - p + s_2)$
No replacement	$v_i Q_\tau + \delta v_i Q_{\tau+1}$	$v_i Q_\tau + \delta v_i Q_{\tau+1}$

Table 1: The utilities of replacing in the first period, second period, and never

Given the utility functions and considering vehicles of age $\tau \geq \eta$, the value for which a consumer is indifferent between replacing her car in the first or the second period is given by

$$v_\tau^{1,2} = \frac{(1-\delta)p + \delta . s_2 - s_1}{\Delta Q_\tau},$$

where $\Delta Q_\tau = Q_0 - Q_\tau$, and the superscript $(1, 2)$ stands for being indifferent between the first and the second periods. Similarly, the value of being indifferent between replacing in the first period and never replacing is given by

$$v_\tau^{1,n} = \frac{p - s_1}{\Delta Q_\tau + \delta . \Delta Q_{\tau+1}},$$

where the superscript $(1, n)$ refers to being indifferent between replacing in period 1 and never replacing.

The government's objective, which will be stated formally later on, is to minimize the subsidy budget, given a target replacement level. The game is played à la Stackelberg, with the government acting as leader. It first announces a subsidy schedule (s_1, s_2), and next, consumers, as followers, react to this announcement by deciding whether or not to replace their vehicles. As is usual in this setting, the equilibrium is obtained by solving the game in reverse order. That is, we start by determining the consumer's reaction to the subsidy schedule and then optimize for the government.

3 Results

In this section, we solve for equilibrium, starting by computing the consumer's reaction function. To save on notation, let

$$\Phi_\tau = \frac{\Delta Q_{\tau+1}}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}}, \qquad \Psi_\tau = \frac{p(\Delta Q_\tau - (1-\delta)\,\Delta Q_{\tau+1})}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}}$$

3.1 Consumer response

Lemma 1 describes the reaction function of consumers owning a car aged τ to any subsidy plan (s_1, s_2) announced by the government.

Lemma 1 *Given any announced subsidy plan, consumer i owning a car aged $\tau \geq \eta$ will replace in the first period if $v_i \geq \chi_\tau(s_1, s_2)$, where*

$$\chi_\tau(s_1, s_2) = \begin{cases} \frac{p-s_1}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}}, & \text{if } s_2 \leq \Phi_\tau s_1 + \Psi_\tau, \\ \frac{(1-\delta)p + \delta s_2 - s_1}{\Delta Q_\tau} & \text{if } s_2 > \Phi_\tau s_1 + \Psi_\tau. \end{cases} \tag{1}$$

Proof. See Appendix. ■

According to Lemma 1, $\chi_\tau(s_1, s_2)$ is either equal to $v_\tau^{1,2}$ or $v_\tau^{1,n}$ depending on the values of s_1 and s_2. When s_2 is lower than the threshold $\Phi_\tau s_1 + \Psi\tau$, then consumers only replace in the first period. Note that this threshold depends on Q_τ, that is, it varies for cars of different ages. Any consumer whose utility for the first-period replacement is higher than no replacement purchases in the first period. The rest of the consumers who have not replaced in the first period would not replace in the second period either. In other words, when s_2 is lower than the above threshold, consumers disregard the option of replacing in the second period, since s_2 is not sufficiently larger than s_1. Therefore, consumers either replace in the first period or do not replace at all. Otherwise, when s_2 is larger than $\Phi_\tau s_1 + \Psi\tau$, consumers can replace in either period. In particular, those who have not replaced their car in the first period do replace in the second period, provided that $v_i \geq \dfrac{p - s_2}{\Delta Q_{\tau+1}}$.

So far, we have characterized the reaction of consumers owning a vehicle aged τ to any subsidy plan. To get a picture of the overall vehicle fleet replacements, we need to account for all cars of various ages. To do so, we investigate how a subsidy plan (s_1, s_2) affects replacement decisions when any two subsequent vintages of cars are considered.

Lemma 2 *Consider two arbitrary subsequent vintages of vehicles, τ and $\tau + 1$, where $\tau \geq \eta$. Assume that Q_τ is a non-increasing convex function in τ and let $p \geq s_1$. Then, $\chi_\tau(s_1, s_2)$ and $\chi_{\tau+1}(s_1, s_2)$ are characterized as follows:*

$$\chi_\tau(s_1, s_2) = \begin{cases} \frac{p-s_1}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}}, & \text{if } s_2 < \Phi_\tau s_1 + \Psi_\tau, \\ \frac{(1-\delta)p+\delta s_2 - s_1}{\Delta Q_\tau}, & \text{if } \Phi_\tau s_1 + \Psi_\tau \leq s_2 \leq \Phi_{\tau+1} s_1 + \Psi_{\tau+1}, \\ \frac{(1-\delta)p+\delta s_2 - s_1}{\Delta Q_\tau}, & \text{if } s_2 > \Phi_{\tau+1} s_1 + \Psi_{\tau+1}, \end{cases}$$

$$\chi_{\tau+1}(s_1, s_2) = \begin{cases} \frac{p-s_1}{\Delta Q_{\tau+1} + \delta.\Delta Q_{\tau+2}}, & \text{if } s_2 < \Phi_\tau s_1 + \Psi_\tau, \\ \frac{p-s_1}{\Delta Q_{\tau+1} + \delta.\Delta Q_{\tau+2}}, & \text{if } \Phi_\tau s_1 + \Psi_\tau \leq s_2 \leq \Phi_{\tau+1} s_1 + \Psi_{\tau+1}, \\ \frac{(1-\delta)p+\delta s_2 - s_1}{\Delta Q_{\tau+1}}, & \text{if } s_2 > \Phi_{\tau+1} s_1 + \Psi_{\tau+1}. \end{cases}$$

Proof. See Appendix. ∎

Lemma 2 shows that when s_2 is lower than the threshold $\Phi_\tau s_1 + \Psi_\tau$, then consumers owning vehicles aged τ and $\tau + 1$ replace only in the first period. If s_2 is larger than this threshold, then some low-value consumers, with vehicles aged τ and who have not replaced in the first period, find it profitable to replace in the second period. However, s_2 is still too low to induce those low-value consumers who have not replaced their vehicle aged $\tau + 1$ in the first period, to replace in the second period. So, they need an even greater subsidy in the second period ($s_2 > \Phi_{\tau+1} s_1 + \Psi_{\tau+1}$) to replace in this period rather than never replacing. This intuitive result stems from the fact that the quality of cars aged $\tau + 1$ is lower than that of cars aged τ. Therefore, in general, consumers who do not replace their car aged $\tau + 1$ in the first period, at level s_1, should have a lower valuation than similar consumers with cars aged τ. Consequently, non-replacers in the first period require a higher subsidy s_2 to prefer replacing in the second period over no replacement at all.

In Lemma 1 and Lemma 2, it is assumed that $\tau \geq \eta$. Since vehicles aged $\eta - 1$ are not eligible for the first-period subsidy, the two lemmas can be rewritten by putting $s_1 = 0$. Therefore, the results and intuitions remain true for $\tau = \eta - 1$ too.

3.2 Government problem

As alluded to before, the government aims at minimizing the subsidy cost, subject to a car-replacement target level Γ. (For clarity, Γ, p, s_1, s_2 and Q take their values in the interval $(0, 1)$ in the theoretical part of the paper. This normalization will be relaxed in the numerical example.) To keep the problem tractable, without much loss of qualitative insight, we make the simplifying assumption that all vehicles aged $\eta - 1$ or older are of low quality, denoted Q_l, and can be replaced by new cars of high quality Q_h. Let $\tilde{Q} = Q_h - Q_l$ be the quality gap between the old and new cars. We assume that $\tilde{Q} \geq p$, which ensures that the added value of a new car is at least equal to the price of the vehicle. The total number of vehicles is normalized to 1 and it is assumed that the numbers of cars with $\tau = \eta - 1$ and $\tau \geq \eta$ are β (≤ 1) and $1 - \beta$, respectively. Based on Lemma 2, we can characterize $\chi_{\eta-1}$ and $\chi_{\geq\eta}$ as follows:

$$\chi_{\eta-1}(s_2) = \begin{cases} \dfrac{p}{(1+\delta)\tilde{Q}}, & \text{if } s_2 < \frac{\delta p}{1+\delta}, \\ \dfrac{p + \delta(-p + s_2)}{\tilde{Q}}, & \text{if } \frac{\delta p}{1+\delta} \leq s_2 \leq \frac{\delta p + s_1}{1+\delta}, \\ \dfrac{p + \delta(-p + s_2)}{\tilde{Q}}, & \text{if } s_2 > \frac{\delta p + s_1}{1+\delta}, \end{cases} \tag{2}$$

$$\chi_{\geq\eta}(s_1, s_2) = \begin{cases} \dfrac{p - s_1}{(1+\delta)\tilde{Q}}, & \text{if } s_2 < \frac{\delta p}{1+\delta}, \\ \dfrac{p - s_1}{(1+\delta)\tilde{Q}}, & \text{if } \frac{\delta p}{1+\delta} \leq s_2 \leq \frac{\delta p + s_1}{1+\delta}, \\ \dfrac{p + \delta(-p + s_2) - s_1}{\tilde{Q}}, & \text{if } s_2 > \frac{\delta p + s_1}{1+\delta}, \end{cases} \tag{3}$$

where $\chi_{\eta-1}$ and $\chi_{\geq\eta}$ refer to vehicles with $\tau = \eta - 1$ and $\tau \geq \eta$, respectively. If $s_2 < \frac{\delta p}{1+\delta}$, then consumers either replace in the first period or do not replace at all. If $\frac{\delta p}{1+\delta} \leq s_2 \leq \frac{\delta p + s_1}{1+\delta}$, then consumers owning cars aged $\eta - 1$ replace in either in the first or second period. However, those who have vehicles aged η or older only replace in the first period. Finally, if $s_2 > \frac{\delta p + s_1}{1+\delta}$, then all car owners may replace either in the first or the second period.

Considering the consumer reaction functions defined above, the government problem is formulated as follows:

$$\begin{aligned} \text{minimize } & (1-\beta)\left(s_1 (1 - \chi_{\geq\eta})^+ + s_2 \left(\chi_{\geq\eta} - \frac{p - s_2}{\tilde{Q}} \right)^+ \right) + \beta \cdot s_2 \left(\chi_{\eta-1} - \frac{p - s_2}{\tilde{Q}} \right)^+ \\ \text{subject to } & (1-\beta)\left((1 - \chi_{\geq\eta})^+ + \left(\chi_{\geq\eta} - \frac{p - s_2}{\tilde{Q}} \right)^+ \right) \\ & + \beta \left((1 - \chi_{\eta-1})^+ + \left(\chi_{\eta-1} - \frac{p - s_2}{\tilde{Q}} \right)^+ \right) \geq \Gamma, s_1, s_2 \geq 0, \end{aligned} \tag{4}$$

where $(x)^+ = \max(x, 0)$. The objective function is composed of the total subsidy paid in both periods for cars with $\tau \geq \eta$ plus the subsidy cost in the second period spent on cars aged $\eta - 1$. The constraint states that the total number of scrapped vehicles must be at least equal to the target level.

The optimization problem in (4) combined with the conditions in (2) and (3) imply that we need to solve three problems. Proposition 1 describes government's optimal policy.

Proposition 1 *The unique equilibrium subsidy plan is given by*

$$(s_1, s_2) = \begin{cases} (0,0) & \text{if } \Gamma < \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}, \\ \left((1+\delta)(\Gamma-1)\tilde{Q}+p,\ (\Gamma-1)\tilde{Q}+p\right), & \text{if } \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \le \Gamma \le \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}, \\ \left(\frac{1}{2}\left((1+\delta)\Gamma-\delta-2\right)\tilde{Q}+p,\ (\Gamma-1)\tilde{Q}+p\right), & \text{if } \Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}. \end{cases} \tag{5}$$

Proof. See Appendix. ■

When the car replacement target level is sufficiently low, that is, $\Gamma < \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, then the government does not need to offer any subsidy for the target to be met. Also, all consumers with a high valuation purchase in the first period only and low-value consumers never replace.

For medium target levels, i.e., $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \le \Gamma \le \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, the subsidy level in both periods is positive and increasing over time. Indeed, we have

$$s_2 - s_1 = \delta(1-\Gamma)\tilde{Q}$$

which is positive. Consumers with cars aged η or older either replace in the first period or do not replace at all. This means that the second-period subsidy is not high enough to induce those who do not replace in the first period to do so in the second period.

Further, although owners of cars aged $\eta - 1$ are not eligible for the subsidy in the first period, some of them can still replace their cars without cashing the subsidy. Now, s_2 is large enough for some of those who were not eligible in period 1 to replace in the second period. In fact, the government is implementing a price- (subsidy-) discrimination policy based on the car's eligibility. One advantage of such a policy is that the subsidy targets consumers with a medium valuation and who do not need a high incentive to replace. This is in line with the government's medium target level. Another advantage is that, by offering a not-too-high second-period subsidy level, high-value consumers do not postpone replacement to free ride on the program.

For $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, the subsidy in either period is increasing in the target level. Now, comparing the subsidy plans for intermediate and high values of the target, we note that the first-period subsidy drops from $(1+\delta)(\Gamma-1)\tilde{Q}+p$ to $\frac{1}{2}\left((1+\delta)\Gamma-\delta-2\right)\tilde{Q}+p$, while the second-period subsidy remains unchanged. Along the same lines as above, as the government needs more owners to replace their cars, it intensifies its subsidy discrimination policy by increasing the differential between the two subsidy levels.

4 An example

In this section, we use an example to illustrate the application of our model in its general form. Here, rather than dividing vehicles into two groups, of old and new cars, we account for all cars of various qualities (ages). In addition, the normalization assumption from the previous section is relaxed, and the data are taken from either the literature or practical cases.

4.1 Parameter setting

Similarly to Zaman and Zaccour (2018), we let $\delta = 0.9$, $\omega = 14$, and $\eta = 10$. Further, to identify the number of vehicles of different ages at the beginning of the first period, we use the density distribution of vehicles aged 1 to 14 provided in Engers et al. (2012). Accordingly, based on the number of vehicles in our setting, we set $\Gamma \in \{20{,}000; 21{,}000; \ldots; 28{,}000\}$. Also as in Zaman and Zaccour (2018), we let the consumer-valuation and government's decision variables have different values, that is,

$$s_1 \in \{0, 100, \ldots, 4,000\}, \quad s_2 \in \{0, 100, \ldots, 4,000\}, \quad \theta \in \{0, 0.1, \ldots, 1.0\}.$$

In addition, we adjust the price and quality of cars as follows:

$$\begin{aligned} p &= \$10,000,\ Q_0 = \$25,000,\ Q_9 = \$20,000,\ Q_{10} = \$15,000,\ Q_{11} = \$10,000, \\ Q_{12} &= \$5,000,\ Q_{13} = \$2,000,\ Q_{14} = \$1,500,\ Q_{15} = \$1,000. \end{aligned}$$

4.2 Numerical results

Considering the above parameter values and ranges, we examine how consumers owning vehicles of different ages react to subsidies. To do so, we fix the first-period subsidy and let the second-period subsidy change. Figure 1 shows the effect of the second-period subsidy on the number of cars aged 9, 10, 11 that are replaced in that period. In Figure 1a, when s_2 is between \$3,000 and \$4,000, while consumers with vehicles aged 9 replace in the second period, owners of cars aged 10 and 11 do not replace. In fact, unlike owners of cars aged 9, who replace either in the first or the second period, those with cars aged 10 and 11 either replace in the first period or do not replace at all. When s_2 goes higher than \$4,000, some of those with cars aged 10 start replacing in the second period too. Cars aged 11 are not replaced in the second period until s_2 reaches \$4,300. Therefore, as mentioned in Lemma 2, older cars are less sensitive to s_2 than are younger cars. A similar trend is observed when the first-period subsidy is fixed to \$1,000 (Figure 1b).

We also demonstrate how the optimal subsidy values change with different levels of Γ. As shown in Figure 2, increasing the target level does not necessarily result in a higher s_1 and s_2. For example, increasing Γ from 25,000 to 26,000 results in a higher s_1 while s_2 remains at the same level. However, when the target level reaches 27,000 units, the government prefers to transfer purchases to the second period by reducing s_1 and increasing s_2 at the same time. This illustrates the idea that increasing the target level may induce the government to lower the first-period subsidy, which is perfectly in line with Proposition 3.

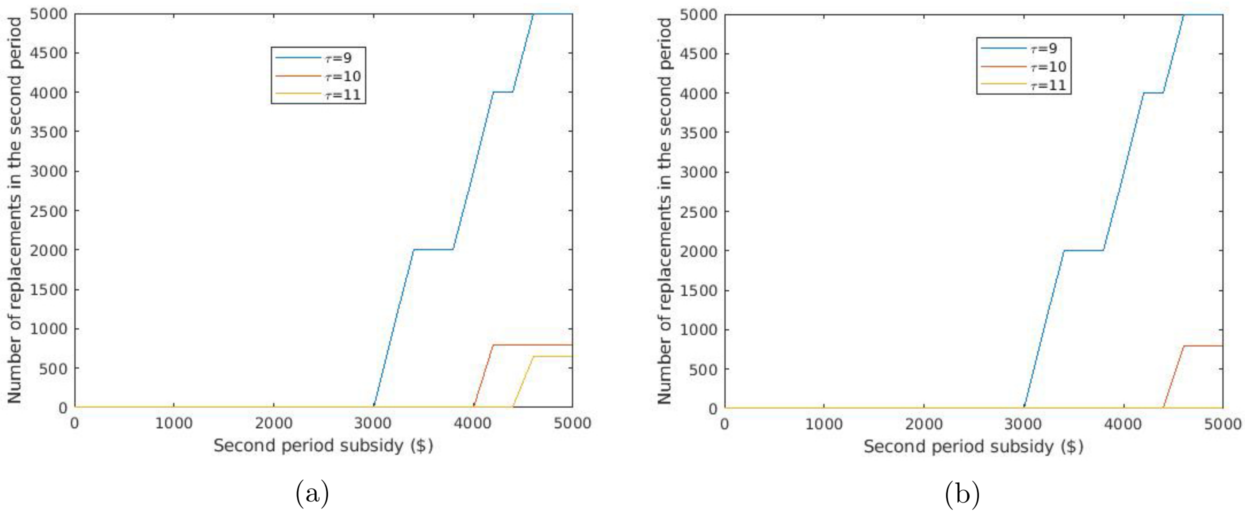

Figure 1: Number of vehicles aged 9, 10, 11 replaced in the second period when s_2 changes from 0 to 5,000 and s_1 is equal to a) 500, and b) 1000.

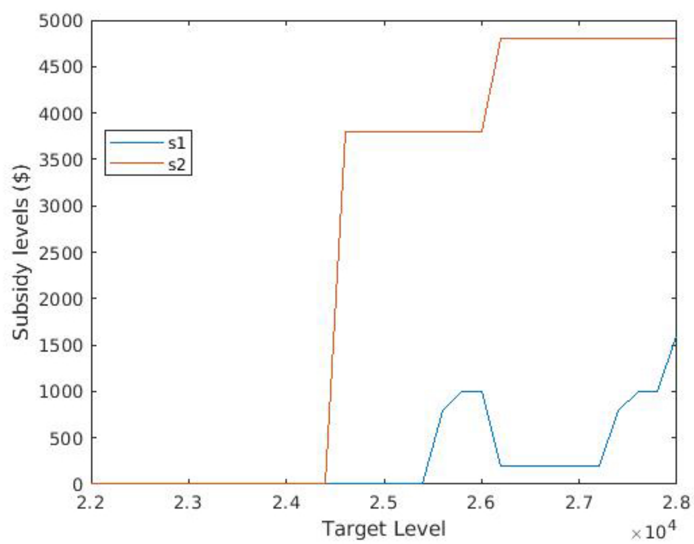

Figure 2: Effect of target level on the first- and second- period subsidy levels

5 Extension: Changing the eligibility age

Increasing the subsidy value enables the government to reach higher target levels. Another potential way to attract more consumers to the subsidy program is to change the eligibility age. To investigate this, we need to do a sensitivity analysis on η. However, since η does not appear explicitly in the model's equations and expressions, the sensitivity analysis on the eligibility age cannot be done by simply taking first-order conditions. Instead, we would check whether the government is better off decreasing the eligibility age from η to $\eta - 1$. Assume that changing the eligibility age from η to $\eta - 1$ makes $\alpha(\leq 1)$ more vehicles eligible for the program. Note that these vehicles are of age $\eta - 2$ and are eligible for the subsidy only in the second period (when they will be aged $\eta - 1$). Also, β vehicles that were previously only eligible in the second period are now eligible in both periods. In summary, $\beta + (1 - \beta)$, that is, all vehicles in the previous setup are now eligible in both periods, and α vehicles are only eligible in the second period. Corollary 1 characterizes the equilibrium in this setting.

Corollary 1 *The unique equilibrium subsidy plan, when the eligibility age is $\eta - 1$ is given by*

$$(s_1, s_2) = \begin{cases} (0,0)\,, & \text{if } \Gamma \leq \frac{\delta\tilde{Q}+\Delta Q-p}{(1+\delta)\tilde{Q}}, \\ \left(\dfrac{(1+\delta)(\Gamma-1)\tilde{Q}+p}{1+\alpha(1+\delta)}, \dfrac{\alpha\delta p+(\Gamma-1)\tilde{Q}+p}{1+\alpha(1+\delta)}\right), & \text{if } \frac{\delta\Delta Q+\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{\delta\Delta Q+(2+\alpha(1+\delta))\left(\tilde{Q}-p\right)}{(1+\delta)\tilde{Q}}, \\ \left(\dfrac{(2+\alpha+\alpha\delta)\left(p-\tilde{Q}\right)+\tilde{Q}\left(\Gamma(1+\delta)-\delta\right)}{2\left(1+\alpha(1+\delta)\right)}, \dfrac{\alpha\delta p+(\Gamma-1)\tilde{Q}+p}{1+\alpha(1+\delta)}\right), & \text{if } \Gamma > \frac{\delta\tilde{Q}+(2+\alpha(1+\delta))\left(\tilde{Q}-p\right)}{(1+\delta)\tilde{Q}}. \end{cases} \tag{6}$$

Proof. The corollary is proved in the same way as Proposition 1. ■

Considering the solutions in (5) and (6), we are interested in characterizing the conditions under which the government is better off reducing the eligibility age. Proposition 2 gives the results.

Proposition 2 *Assume that the government can choose between ages η and $\eta - 1$ for a vehicle to be eligible for the scrappage program. The optimal policy of the government for different target levels is as follows:*

1. *When* $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, *then*

$$\begin{array}{lr} \textit{if } \alpha \geq \hat{\alpha} & \eta - 1 \textit{ is superior} \\ \textit{if } \alpha < \hat{\alpha} \textit{ and } \beta \geq \hat{\beta} & \eta \textit{ is superior} \\ \textit{if } \alpha < \hat{\alpha} \textit{ and } \beta < \hat{\beta} \textit{ and } \begin{cases} \Gamma > \hat{\Gamma} & \eta - 1 \textit{ is superior} \\ \Gamma \leq \hat{\Gamma} & \eta \textit{ is superior} \end{cases} & \end{array}$$

where $\hat{\alpha}$, $\hat{\beta}$ *and* $\hat{\Gamma}$ *are given by*

$$\begin{aligned} \hat{\alpha} &= -\frac{\beta(1+\delta)(p-\tilde{Q})}{(\tilde{Q}+\beta(p-\tilde{Q}))\delta^2 + (1+2\delta)(\beta-1)(p-\tilde{Q})}, \\ \hat{\beta} &= \frac{\alpha\left((2+2\alpha+4\delta+5\alpha\delta+3\alpha\delta^2)\left(p-\tilde{Q}\right) - \tilde{Q}\delta^2\left(1+\alpha\left(1+\delta\right)\right)\right)}{(1+\delta)(p-\tilde{Q})(1+\alpha(1+\delta))^2}, \\ \hat{\Gamma} &= \frac{\alpha\delta\left(p\alpha+\tilde{Q}\right) - \beta\left(1+\alpha\left(1+\delta\right)\right)^2\left(p-\tilde{Q}\right)}{\alpha\tilde{Q}(1+2\delta+\alpha(1+\delta)^2)} \end{aligned}$$

2. *When* $\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} + \frac{\alpha\left(\tilde{Q}-p\right)}{\tilde{Q}}$, *there is a unique* $0 < \check{\alpha} < 1$ *where:*

$$\begin{cases} \textit{if } \alpha \geq \check{\alpha} & \eta - 1 \textit{ is superior} \\ \textit{if } \alpha < \check{\alpha} & \eta \textit{ is superior.} \end{cases}$$

3. *When* $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} + \frac{\alpha\left(\tilde{Q}-p\right)}{\tilde{Q}}$, $\eta - 1$ *is always superior.*

Proof. See Appendix. ■

Proposition 2 indicates that the comparison between the eligibility ages η and $\eta-1$ depends on all parameter values, and in particular on α, which measures the incremental number of vehicles that become eligible, and on the target Γ. In a nutshell, depending on the target level, there is a minimum threshold on α after which it is cost efficient to decrease the eligibility age from η to $\eta - 1$. In particular, for a target level satisfying

$$\frac{(1+\delta)\tilde{Q} - p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q} - 2p}{(1+\delta)\tilde{Q}},$$

when $\alpha \geq \hat{\alpha}$, $\eta - 1$ is superior to η. Otherwise, the result is ambiguous and depends on the values of Γ and β. When Γ increases to higher levels, that is,

$$\frac{\delta\tilde{Q} + 2\left(\tilde{Q} - p\right)}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{\delta\tilde{Q} + 2\left(\tilde{Q} - p\right)}{(1+\delta)\tilde{Q}} + \alpha\frac{\tilde{Q} - p}{\tilde{Q}},$$

again there is a threshold for α after which the government is better off when changing η to $\eta - 1$. In this case, for α less than this threshold, the government always sticks to η. Finally, when

$$\Gamma > \frac{(2+\delta)\,\tilde{Q} - 2p}{(1+\delta)\tilde{Q}} + \frac{\alpha\left(\tilde{Q} - p\right)}{\tilde{Q}},$$

$\eta - 1$ would always satisfy the target level at a lower cost than η.

6 Conclusion

In this paper, we use a two-period game framework to find the optimal vehicle scrappage subsidy policies offered to strategic consumers. We obtain that the government gives a higher subsidy in the second than the first period to price-discriminate between high-value and low-value consumers. Low-value consumers will not replace in the second period unless the subsidy in that period is high enough compared to that of the first period. We show that low-value consumers with older cars are more reluctant to replace their car in the second period than those owning younger cars. We also demonstrate that both subsidy levels are increasing in the government's car replacement target level . However, after a target level threshold, the first-period subsidy drops to a lower level once, while the second-period subsidy keeps increasing.

We also study the eligibility age set by the scrappage program from a parameter-design point of view. In particular, the policy maker may be better off decreasing the eligibility age of the subsidy program. According to our results, this depends on the number of new vehicles becoming eligible with a lower eligibility age and on the target level.

Two challenging extensions to our paper are worth looking. First, while in this work, we only focus on the interaction between the government and consumers, it would be also interesting to consider the strategic role of the manufacturer. Indeed, the manufacturer may adjust her prices in the presence of a subsidy program, thereby affecting the pass-through effect of the subsidy. Here, one important question to look at is whether the government should subsidize the consumer, the manufacturer, or both? Second, since one of the main aims of vehicle scrappage programs is to reduce car pollution, the environmental effect of the subsidy policies needs to be taken into account. Therefore, it is worth attempting to either analyze the environmental effect of the policies ex-post or account for them directly in the government objective.

A Appendix

Proof of Lemma 1. If $v_\tau^{1,2} \leq v_\tau^{1,n}$, that is,

$$s_2 \leq s_1 \left(\frac{\Delta Q_{\tau+1}}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}} \right) + \frac{p(\Delta Q_\tau + \delta.\Delta Q_{\tau+1} - \Delta Q_{\tau+1})}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}},$$

then consumers either replace in the first period or never replace. So, $\chi_\tau(s_1, s_2) = v_\tau^{1,n} = \frac{p-s_1}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}}$. If $v_\tau^{1,2} \geq v_\tau^{1,n}$, that is,

$$s_2 \geq s_1 \left(\frac{\Delta Q_{\tau+1}}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}} \right) + \frac{p(\Delta Q_\tau + \delta.\Delta Q_{\tau+1} - \Delta Q_{\tau+1})}{\Delta Q_\tau + \delta.\Delta Q_{\tau+1}},$$

then consumers with a valuation higher than $\chi_\tau(s_1, s_2)$ replace in the first period, and the rest go on to the second period to make their replacement decision. So,

$$\chi_\tau(s_1, s_2) = v_\tau^{1,2} = \frac{(1-\delta)p + \delta.s_2 - s_1}{\Delta Q_\tau}.$$

Proof of Lemma 2. In order to prove this lemma, we need to show that

$$s_1\left(\frac{\Delta Q_{\tau+1}}{\Delta Q_\tau+\delta.\Delta Q_{\tau+1}}\right)+\frac{p(\Delta Q_\tau+\delta.\Delta Q_{\tau+1}-\Delta Q_{\tau+1})}{\Delta Q_\tau+\delta.\Delta Q_{\tau+1}}\leq s_1\left(\frac{\Delta Q_{\tau+2}}{\Delta Q_{\tau+1}+\delta.\Delta Q_{\tau+2}}\right)+\frac{p(\Delta Q_{\tau+1}+\delta.\Delta Q_{\tau+2}-\Delta Q_{\tau+2})}{\Delta Q_{\tau+1}+\delta.\Delta Q_{\tau+2}}.$$

To do so, it suffices to show that the right-hand side (RHS) of this inequality is increasing in τ. The first derivative of the RHS is given by

$$\frac{(p-s_1)\left(\frac{\partial\Delta Q_\tau}{\partial\tau}\cdot\Delta Q_{\tau+1}-\frac{\partial\Delta Q_{\tau+1}}{\partial\tau}\cdot\Delta Q_\tau\right)}{(\Delta Q_\tau+\delta.\Delta Q_{\tau+1})^2}. \tag{7}$$

Under the assumptions that Q_τ is a non-increasing convex function in τ and $p\geq s_1$, the above derivative is clearly non-negative.

Proof of Proposition 1. We need to solve the following three problems:

$$\begin{aligned}
\textbf{P1} &: \min(1-\beta)s_1\left(1-\frac{p-s_1}{(1+\delta)\tilde{Q}}\right),\\
\text{subject to} &:\\
\Gamma &\leq 1-\frac{p-s_1}{(1+\delta)\tilde{Q}},\\
s_2 &\leq \frac{\delta p}{1+\delta},\\
s_1,s_2 &\geq 0.
\end{aligned} \tag{8}$$

$$
\begin{aligned}
\mathbf{P2} \quad &: \quad \min\left\{(1-\beta)\left(s_1\cdot\left(1-\frac{p-s_1}{(1+\delta)\tilde{Q}}\right)+s_2\cdot\left(\frac{p-s_1-(1+\delta)\,(p-s_2)}{(1+\delta)\tilde{Q}}\right)\right)\right. \\
&\qquad \left. +\,\beta\cdot s_2\cdot\left(\frac{p-(1+\delta)\,(p-s_2)}{\tilde{Q}}\right)\right\} \\
\text{subject to} \quad &: \\
\Gamma &\le 1-\frac{p-s_2}{\tilde{Q}}, \qquad (9)\\
\frac{\delta p}{1+\delta}-s_2 &\le 0, \\
s_2-\frac{\delta p+s_1}{1+\delta} &\le 0, \\
s_1, s_2 &\ge 0.
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P3} \quad &: \quad \min\left\{(1-\beta)\left(s_1\cdot\left(1-\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}\right)+s_2\cdot\left(\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right)\right)\right. \\
&\qquad \left. +\beta\cdot s_2\cdot\left(\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\right)\right\}, \\
1-\frac{p-s_2}{\tilde{Q}} &\ge \Gamma, \qquad (10)\\
s_2 &\ge \frac{\delta p+s_1}{1+\delta}, \\
s_1, s_2 &\ge 0.
\end{aligned}
$$

In the following, we provide the solution of each problem.

Problem **P1** : The objective function of (8) is increasing and convex in s_1 and independent of s_2. The first constraint is increasing in s_1 and independent of s_2. The second constraint is increasing in s_2 and independent of s_1. These conditions imply that the optimal solution of (8) is achieved when the first constraint is met. The solutions are as follows:

$$
s_1=\frac{(1+\delta)(\Gamma-1)\tilde{Q}+p}{1-\beta},\ s_2=0,\ \text{if } (1+\delta)(\Gamma-1)\tilde{Q}+p\ge 0\ \left(\text{i.e., } \Gamma\ge\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\right). \quad (11)
$$

When the condition in (11) is not satisfied, i.e., $\Gamma \le \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, both subsidy values are zero.

Problem **P2** : The Lagrangian function is as follows:

$$\mathcal{L}(s_1,s_2,\mu_1,\mu_2,\mu_3)=(1-\beta)\Bigg(s_1\cdot(1-\frac{p-s_1}{(1+\delta)\tilde{Q}})+s_2\cdot(\frac{p-s_1}{(1+\delta)\tilde{Q}}-\frac{p-s_2}{\tilde{Q}})\Bigg)$$
$$+\beta\cdot s_2\cdot\Bigg(\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\Bigg)+\mu_1\cdot(\Gamma-(1-\frac{p-s_2}{\tilde{Q}}))$$
$$+\mu_2\cdot\Bigg(\frac{\delta p}{1+\delta}-s_2\Bigg)+\mu_3\cdot\Bigg(s_2-(\frac{\delta p+s_1}{1+\delta})\Bigg) \tag{12}$$

Considering Kuhn-Karush-Tucker conditions, the feasible solutions are as follows:

$$\mu_1=0,\ \mu_2=\frac{-(1+\delta)(-1+\beta)\tilde{Q}+p(-1+(1+\delta)\beta)}{\tilde{Q}},$$
$$\mu_3=-\frac{(-+\beta)(\delta\cdot\tilde{Q}-p+\tilde{Q})}{\tilde{Q}},\ s_1=0,\ s_2=\frac{\delta\cdot p}{1+\delta},$$
$$\text{If } (1+\delta)(\Gamma-1)\tilde{Q}+p<0\ \ \Bigg(\text{i.e., } \Gamma\leq\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\Bigg).$$

$$\mu_1=-(1+\delta)(\beta-2\Gamma+1)\tilde{Q}+p\cdot(\beta\cdot\delta+\beta+1),\ \mu_2=0,\mu_3=-\frac{\beta\cdot(1+\delta)(-1+\beta)(p-\tilde{Q})}{\tilde{Q}},$$
$$s_1=(1+\delta)(\Gamma-1)\tilde{Q}+p,\ s_2=\Gamma\cdot\tilde{Q}+p-\tilde{Q}$$
$$\text{If } (1+\delta)(\Gamma-1)\tilde{Q}+p\geq 0\ \ \Bigg(\text{i.e., } \Gamma\geq\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}\Bigg). \tag{13}$$

Problem **P3** : The Lagrangian function is as follows:

$$\mathcal{L}(s_1,s_2,\mu_1,\mu_2)=(1-\beta)\Bigg(s_1\cdot\Bigg(1-\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}\Bigg)+s_2\cdot\Bigg(\frac{p+\delta(-p+s_2)-s_1}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}}\Bigg)\Bigg)$$
$$+\beta\cdot s_2\cdot(\frac{p+\delta(-p+s_2)}{\tilde{Q}}-\frac{p-s_2}{\tilde{Q}})$$
$$+\mu_1\cdot(\Gamma-(1-\frac{p-s_2}{\tilde{Q}}))$$
$$+\mu_2\cdot\Bigg((\frac{\delta p+s_1}{1+\delta})-s_2\Bigg) \tag{14}$$

From the Kuhn-Karush-Tucker conditions, the only feasible solution is

$$\mu_1 = \frac{1}{2}(1+\delta)\Big((\Gamma-1)(-1+\beta)\cdot\delta + (\Gamma-2)\cdot\beta + 3\Gamma - 2\Big)\cdot\tilde{Q} + p\cdot(\beta\cdot\delta+\beta+1),\ \mu_2 = 0,$$
$$s_1 = \frac{1}{2}\Big((1+\delta)\cdot\Gamma - \delta - 2\Big)\cdot\tilde{Q} + p,\ s_2 = \Gamma\cdot\tilde{Q} + p - \tilde{Q}$$
$$\text{IF } \frac{1}{2}\Big((1+\delta)\cdot\Gamma - \delta - 2\Big)\cdot\tilde{Q} + p \geq 0\ \left(\text{i.e., } \Gamma > \frac{(2+\delta)\tilde{Q} - 2p}{(1+\delta)\tilde{Q}}\right). \tag{15}$$

We compare the subsidy cost for the three regions of Γ.

- When $\Gamma < \frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, then $s_1 = s_2 = 0$ is the optimal subsidy.
- When $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \leq \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, the difference between the cost in (11) and (13) is given by
$$\frac{\beta\cdot((1+\delta)(\Gamma-1)\tilde{Q}+p)(-(1+\delta)(\beta-\Gamma)\tilde{Q} + ((-1+\beta)\cdot\delta+\beta)p)}{(-1+\beta)(1+\delta)\tilde{Q}},$$
which is non-negative for $\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} \leq \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$. Therefore, the optimal subsidy levels are
$$s_1 = (1+\delta)(\Gamma-1)\tilde{Q} + p, \quad s_2 = \Gamma\cdot\tilde{Q} + p - \tilde{Q}.$$
- When $\Gamma > \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, the difference between the cost in (13) and (15) is given by
$$\frac{1}{4}(1-\beta)\cdot\tilde{Q}\cdot(\Gamma\cdot(1+\delta)-\delta)^2,$$
which is non-negative. Therefore, the optimal subsidy levels are
$$s_1 = \frac{1}{2}\Big((1+\delta)\cdot\Gamma - \delta - 2\Big)\cdot\tilde{Q} + p,\ s_2 = \Gamma\cdot\tilde{Q} + p - \tilde{Q}.$$

Proof of Proposition 2. We need to compare the subsidy costs in (5) and (6) for each region.

- When
$$\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}},$$
the difference between the costs in (5) and (6) is given by
$$\frac{(1+\delta)(\Gamma-1)\tilde{Q}+p}{(1+(1+\delta)\cdot\alpha)^2\cdot\tilde{Q}}\{(-(1+\delta)^2(\beta-\Gamma)\Delta Q+p\cdot(\beta\cdot\delta^2+(2\beta-1)\cdot\delta+\beta))\cdot\alpha^2$$
$$+(((-2\beta+2\Gamma-1)\cdot\delta-2\beta+\Gamma)\tilde{Q}+2p\cdot\beta\cdot(1+\delta))\cdot\alpha+\beta\cdot(p-\Delta Q)\Big\},$$
which is the product of two expressions. The first expression is always non-negative. The first derivative of the second expression with respect to Γ is
$$(1+\delta)^2\cdot\tilde{Q}\cdot\alpha^2+(2\delta+1)\cdot\tilde{Q}\cdot\alpha,$$
which is non-negative. Also, $\Gamma=\hat{\Gamma}$ is the root of the second expression. When $\alpha\geq\hat{\alpha}$, $\hat{\Gamma}\leq\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}}$, that is, the cost difference is positive and $\eta-1$ is superior. When $\alpha<\hat{\alpha}$ and $\beta\geq\hat{\beta}$, $\hat{\Gamma}\geq\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}}$, that is, the cost difference is negative and η is superior. When $\alpha<\hat{\alpha}$ and $\beta<\hat{\beta}$,
$$\frac{(1+\delta)\tilde{Q}-p}{(1+\delta)\tilde{Q}} < \hat{\Gamma} \leq \frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}},$$
that is, when $\Gamma>\hat{\Gamma}$, $\eta-1$ is superior and when $\Gamma\leq\hat{\Gamma}$ η is superior.

- When
$$\frac{(2+\delta)\tilde{Q}-2p}{(1+\delta)\tilde{Q}} < \Gamma \leq \frac{((1+\delta)\alpha+\delta+2)\tilde{Q}-(2+(1+\delta)\alpha)p}{(1+\delta)\tilde{Q}},$$

the difference between the costs in (5) and (6) is as follows:

$$
\begin{aligned}
f(\alpha) \quad & \triangleq \frac{1}{4}(-1+\beta)\Big(((\Gamma-1)^2\cdot\delta^2+(-2\Gamma^2+2\Gamma)\cdot\delta-3\Gamma^2+4\Gamma)\tilde{Q}-4\Gamma\cdot p\Big) \\
& +\frac{\Big((1+\delta)(\Gamma-1)\cdot\tilde{Q}+p\Big)\Big((\Gamma-1)\tilde{Q}+p\Big)\beta}{\tilde{Q}} \\
& -\frac{\Big(((\Gamma+\delta)\cdot\alpha+\Gamma)\tilde{Q}+p\cdot\alpha^2\cdot\delta\Big)\Big((1+\delta)(\Gamma-1)\cdot\tilde{Q}+p\Big)}{\tilde{Q}\Big(1+(1+\delta)\cdot\alpha\Big)^2}.
\end{aligned}
\tag{16}
$$

The function $f(\alpha)$ is continuous and its first derivative is positive, that is, (16) is increasing in α (for this range of Γ). Also, (16) is negative for $\alpha=0$ and positive for $\alpha=1$. Therefore, there is a unique $\alpha=\breve{\alpha}$ that makes (16) equal to zero, where

$$
\begin{aligned}
\breve{\alpha} = & \frac{-4(\frac{1}{2}\tilde{Q}(\Gamma-1)\delta+\frac{1}{2}\tilde{Q}.\Gamma+p-\tilde{Q})^2(1+\delta)\beta+\tilde{Q}^2(\Gamma-1)^2\delta^3-\tilde{Q}^2(\Gamma-1)^2\delta^2}{4(\frac{1}{2}\tilde{Q}(\Gamma-1)\delta+\frac{1}{2}\tilde{Q}.\Gamma+p-\tilde{Q})^2(1+\delta)^2\beta} \\
& \frac{-4(\frac{3}{4}\Gamma^2.\tilde{Q}+(p-\frac{3}{2}\tilde{Q})\Gamma-\frac{1}{2}p+\frac{1}{2}\tilde{Q})\tilde{Q}.\delta-\Gamma^2.\tilde{Q}^2-2\tilde{Q}(p-\tilde{Q})\Gamma}{-\tilde{Q}^2(\Gamma-1)^2\delta^4+2\tilde{Q}^2(\Gamma-1)\delta^3} \\
& \frac{+4((\frac{1}{4}(\delta(1+\delta)^2\beta-\delta^3+2\delta^2+3\delta+1)\tilde{Q}^2.\Gamma^2+\delta((-\frac{1}{2}\delta.\tilde{Q}+p-\tilde{Q})(1+\delta)\beta}{+(6\Gamma^2.\tilde{Q}^2-6\Gamma.\tilde{Q}^2+4p.\tilde{Q}-\tilde{Q}^2)\delta^2} \\
& \frac{+\frac{1}{2}\tilde{Q}(\delta^2-\delta-1))\tilde{Q}.\Gamma+\delta((-\frac{1}{2}\delta.\tilde{Q}+p-\tilde{Q})^2\beta-\frac{1}{4}\delta.\tilde{Q}^2(\delta-1))}{+(8\Gamma^2.\tilde{Q}^2+(4p.\tilde{Q}-10\tilde{Q}^2)\Gamma-4p(p-\tilde{Q}))\delta} \\
& \frac{(\tilde{Q}(1+\delta)\Gamma-\delta.\tilde{Q}+p-\tilde{Q})^2)^{\frac{1}{2}}}{+4(\frac{3}{4}\tilde{Q}.\Gamma+p-\tilde{Q})\Gamma.\tilde{Q}}
\end{aligned}
$$

- When

$$
\Gamma > \frac{((1+\delta)\alpha+\delta+2)\tilde{Q}-(2+(1+\delta)\alpha)p}{(1+\delta)\tilde{Q}},
$$

the difference between the costs in (5) and (6) is as follows:

$$
\frac{1}{4}\frac{\Big((1+\delta)^2(2(-1+\beta)\delta^2+4(\beta+1)\cdot\delta+2\beta+6)\cdot\alpha^2+2(1+\delta)(2+(-1+\beta)\delta^2+4(\beta+1)\cdot\delta+2\beta+2)\cdot\alpha+2(1+\delta)^2\beta\Big)\tilde{Q}}{(1+\delta)^2\alpha^2},
$$

which is a positive value.

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