Strategic Climate Policies with Endogenous Plant Location: The Role of Border Carbon Adjustments

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Abstract

The relocation of firms is one of the main concerns of governments when designing their climate policies. In a strategic trade model with endogenous plant location, this paper studies the effect of border carbon adjustments (BCAs) on an emission tax competition game between two asymmetric countries. We consider two policy games: a simultaneous and a sequential game associated with the notions of Nash equilibrium and Stackelberg equilibrium, respectively. Without BCAs, a 'race to the bottom' is the unique Nash equilibrium, with high global emissions. In a Stackelberg equilibrium, a second less negative outcome may emerge, which constitutes a Pareto-improvement for both countries compared to the Nash equilibrium provided marginal damages of the Stackelberg leader are sufficiently large. Adding BCAs to the policy game also reduces the pressure on the 'race to the bottom' in carbon taxes, even though socially optimal welfare levels are not obtained. We show that a pure strategy Nash equilibrium with BCAs may not exist but exists if it is really needed, i.e., if global marginal damages are large and countries are highly asymmetric in terms of the perception of environmental damages. In such cases, BCAs will lead to more stringent climate policies in both countries associated with a larger global welfare level. In the sequential game, BCAs not only prevent total relocation of the home firm of the environmentally more concerned country, but may also act as a threat to attract the foreign firm.

Keywords: Endogenous Plant Location, Global Emissions, Emission Tax Competition, Border Carbon Adjustments

JEL-Classification: R3, F18, F12, Q58, H23, H87

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1 Introduction

The history of climate change negotiations suggests that there is a lack of coordinated actions to reduce greenhouse gases. Global action is difficult due to strong free-rider incentives and sub-global actions are undermined by 'carbon leakage', i.e., emission reductions by some environmentally friendly countries are partly or completely offset by higher emissions in environmentally less concerned countries. One important channel of carbon leakage is through the relocation of production of emission-intensive industries to countries with laxer environmental policies to avoid higher production costs. The relocation of production may not only imply that production partially shifts, but even that emission-intensive industries completely close down and relocate their production facilities abroad. This phenomenon has been referred to as the 'pollution haven hypothesis'(PHH). Although there is mixed empirical evidence of the PHH, the threat of relocation is frequently used by lobbying groups against stricter environmental regulations.[1] Most fears associated with the PHH are the loss of jobs and investment. For this reason, governments may be inclined to implement laxer environmental policies in an attempt to keep firms or to even attract foreign firms, counterveiling global efforts to reduce greenhouse gas emissions significantly.

Recently, border carbon adjustments (BCAs) have been proposed to mitigate carbon leakage and to supplement more stringent greenhouse gas policies. BCAs may take the form of an import tariff or an export rebate or both. Apart from being a measure to protect domestic industries facing a more stringent environmental policy from foreign competition, many scholars argue that BCAs have a strategic effect in either pushing other countries to adopt higher carbon taxes as well, either as an optimal response in a non-cooperative game or by making them to join a climate agreement in order to avoid negative consequences of high tariffs. However, even in the absence of strategic aspects, and adopting a pure welfare perspective, introducing trade measures to complement environmental policies can be justified as already demonstrated by Markusen (1975). He shows that in the absence of global action, the optimal combination of policies to internalise a global pollutant is a Pigouvian tax and an import tariff.[2] In this sense, BCAs are proposed to correct distortions resulting from uneven carbon policies and hence are not considered as disguised trade barriers (Helm et al., 2012).

In this paper, we are interested to find out whether and under which conditions BCAs can support the implementation of more ambitious climate policies by taking into account that firms cannot only relocate parts of their production but even their entire production facilities abroad (endogenous plant location) and that gov-

[1] For example, Eskeland and Harrison (2003) and Manderson and Kneller (2012), among others, find no evidence of the PHH. In contrast, Fredriksson et al. (2003), Xing and Kolstad (2002) and Kellenberg (2009) report significant effects of environmental policies on the location of firms.

[2] See also Hoel (1996) and Copeland (1996) for similar results. These models assume perfect competition.

ernments engage in a strategic emission tax competition game (bilateral and endogenous policy choices) which may perceive global damages from greenhouse gases differently (asymmetric countries). Therefore, our paper is related to two strands of the strategic environmental-trade policy literature, which all build on the strategic imperfect-competition trade model due to Brander and Spencer (1985) by adding environmental damages in governments´ welfare function. The first strand studies strategic environmental policies assuming fixed plant location (immobile firms). This includes for instance, Conrad (1993), Barrett (1994) and Kennedy (1994). This literature concludes for Cournot competition that if environmental policy is the only instrument available to governments, environmental taxes are set below marginal damages because governments have an incentive to give their firms a strategic advantage over their rivals. An important extension are the papers by Eyland and Zaccour (2014), Anouliés (2015)and Baksi and Chaudhuri (2017). These papers analyse how BCAs affect the strategic choice of environmental policies of two countries. Eyland and Zaccour (2014) show, based on numerical simulations, that adding BCAs allows countries to set higher carbon taxes in the non-cooperative equilibrium. Both Anouliés (2015) and Baksi and Chaudhuri (2017) focus on the role of BCAs as a threat to enforce cooperation among asymmetric countries. They show that BCAs may reduce the welfare level of the less environmentally concerned country sufficiently enough such that this makes cooperation more likely to be achieved.

The second strand of literature allows for mobile firms/plants, and focuses on the effect of environmental policies on the location choice of firms. Two types of models have emerged: the location and the market share game where the difference lies in the sequence of the game. The location game assumes that governments move first and then firms choose their location, also called ex-ante policy game. In contrast, in the market share game, firms choose first their location and then governments choose their policies, also called ex-post policy game.[3] By construction, governments cannot affect the location of their firms in the market share game and hence these type of games are less interesting for our analysis.[4]

Some of the early studies which analyse the effect of environmental policies on the location decisions of firms include for instance Markusen et al. (1993) and Motta and Thisse (1994). However, these papers, like the more recent paper by Sanna-Randaccio et al. (2017), assume an exogenous policy level, and hence ignore the effect of firm mobility on the incentives of countries to set their environmental policies. The simplest extension to address this shortcoming are plant location games that assume a unilateral climate policy, as for instance in Petrakis and Xepapadeas (2003), Ikefuji et al. (2010). However, governments do not choose their policies in isolation, which requires an endogenous bilateral policy choice like in Markusen et al. (1995), Rauscher (1995), Hoel (1997), and Ulph and Valentini (2001). For instance, Markusen et al.

[3] See for instance Ulph and Valentini (2001) and Petrakis and Xepapadeas (2003) for a detailed comparison between the two tpye of games.

[4] For examples of market share type games, see Eerola (2006), De Santis and Stähler (2009) and Dijkstra et al. (2011).

(1995) conclude that there are mainly two possible Nash equilibria, which depend on the level of the damage from pollution: very low environmental regulations, i.e. 'race to the bottom', or very strict environmental regulations that might lead firms to exit the market, i.e., race to the top or ' not-in-my backyard'. Both Rauscher (1995) and Hoel (1997) simplify the assumptions made by Markusen et al. (1995) by ignoring transportation and set up cost and obtain similar qualitative results.

All of the papers belonging to the second strand of literature do not consider BCAs and most assume local pollution [5], a monopolistic market structure or both, of which neither of these assumptions are useful in our context. As mentioned above, BCAs are proposed to achieve two objectives. Firstly, they aim at addressing a global externality (environmental objective) and secondly they aim at reducing leakage effects by protecting domestic industries by levelling the playing field for domestic and foreign firms (competitiveness objective). Therefore, we need to consider a global pollutant, implying that a not-in-my-backyard motivation cannot be rational for governments. In fact, for an environmentally friendly government the motivation may just be reversed: keep-firms-in-my-backyard as there is no point of loosing a firm if environmental taxes are lower abroad. Secondly, we assume an oligopolistic market structure in order to capture the competitiveness objective.

We model an emission tax competition between two governments which strive to attract the plants of two firms, which produce a homogeneous emission-intensive good and compete in a Nash-Cournot fashion in an intra-industry trade model. Countries evaluate damages from global emissions differently. We solve our location game under two different policy regimes. Under the No-BCA regime, each government imposes a carbon tax on the production of those plants which locate within its national boundaries. Under the BCA- regime, the country that sets a higher carbon tax can additionally impose BCAs on imports from plants located in the foreign country. As mentioned by Markusen et al. (1995), endogenous plant location may lead to a discontinuity of welfare functions with respect to tax levels as firms may change their location abruptly above a threshold tax. In our model, this leads to non-continuous reaction functions under the BCA-regime, which implies that Nash equilibria may not exist. Therefore, as Stackelberg equilibria always exist, we also consider the possibility that governments choose their taxes sequentially. Apart from this technical point, a sequential policy choice also gives rise to new interesting results.

We find that if countries choose their policies simultaneously, we end up in a 'race to the bottom' under the No-BCA regime. Different from papers that assume local pollution, this is the unique pure strategy Nash equilibrium in our model, irrespective of the degree of asymmetry among countries. Instead, by moving sequentially, the Stackelberg leader may be able to avoid being stuck at the bottom by imposing a higher carbon tax than the follower, where the latter will attract all plants in this

[5] Rauscher (1995) considers in one case the possibility of transboundary pollution.

equilibrium. This Stackelberg equilibrium is Pareto-improving for both countries and leads to lower global emissions than the Nash equilibrium.

Under the BCA-regime, we show that a Nash equilibrium may not exist, but it exists when it is really needed, i.e., if global marginal damages are high and countries are sufficiently asymmetric. In such cases, the BCA-policy is an effective measure to eliminate the 'race to the bottom', resulting in higher global welfare, though global welfare falls short of the social optimum. Moreover, and similar to models with fixed plant location, BCAs allow both countries to impose stricter climate policies, leading to lower global emissions. Possible equilibria entail partial relocation of plants from the more to the less environmentally concerned country.

The effect of BCAs on Stackelberg equilibria depend greatly on the identity of the leader and the parameters values of our model, representing market size and absolute as well as relative environmental damage evaluation in the two countries. However, one common result is that a 'race to the bottom' becomes less likely to emerge as an equilibrium. Furthermore, we show that if global marginal damages are high and individual damage evaluations are very different in the two countries, BCAs always reduce global emissions.

The remainder of the paper is organised as follows. In Section 2, we present the model. In section 3 and 4, we solve the first stage of our game and derive the climate policy equilibria under the two alternative policy regimes. In Section 5, we compare equilibrium outcomes in order to show the effect of BCAs. Section 6 concludes and discusses possible future research.

2 Model

We present first the basic set up of the model, after which we discuss its main features and assumptions. Then we briefly comment on possible location equilibria before finally deriving the socially optimal solution as a normative benchmark, against which we compare non-cooperative equilibria in our subsequent analysis.

2.1 Basic Ingredients

We consider two countries, respectively, their governments, $i = 1, 2$, which interact in a strategic trade model. The game unfolds in three stages, which is solved by backwards induction. In the first stage, governments choose their policy levels, in the second stage firms choose their location and in the third stage firms choose their outputs.

In the last stage, there are two firms, $k = 1, 2$, which produce a homogeneous good x and compete in outputs, i.e., in Cournot-fashion. Firm 1 is initially located in country 1, and firm 2 is initially located in country 2. Each firm has two plants, one plant supplying the home market in country i, the other plant supplying the foreign market j. That is, markets are segmented. The inverse demand function in market i is given by:

$$p_i(X_i) = a - X_i \quad \forall\ i = 1, 2 \tag{1}$$

where p_i is the market price and parameter $a > 0$ is the chock-off price. $X_i = x_{1i} + x_{2i}$ is total consumption in country i where x_{1i} and x_{2i} are the outputs supplied by firm 1 and 2 to market i, respectively. For simplicity, we assume completely identical firms with a linear production cost function, i.e., $C_{ki}(x_{ki}) = cx_{ki}$ for $k = 1, 2$ and $i = 1, 2$.

Good x is an emission intensive good, like cement or steel, which generates greenhouse gas emissions, for example due to its use and combustion of energy in the production process. Without loss of generality, we assume a constant emission-output ratio across firms, which we normalise to 1, such that an emission tax is de facto an output tax. Hence, profits of firm 1 and 2 obtained in market i are given by

$$\pi_{1i} = (p_i(X_i) - c - t_{1i})x_{1i} \ \&\ \pi_{2i} = (p_i(X_i) - c - t_{2i})x_{2i} \ \forall\ i = 1, 2 \tag{2}$$

where t_{1i} is the effective tax which firm 1 faces on its supply to market i and t_{2i} is the effective tax which firm 2 faces on its supply to market i.

We consider two regimes: the No-BCA-regime and the BCA-regime. In order to illustrate the difference between the two regimes, suppose firm k produces with one of its plants for market i. Following our arguments above, this plant faces an effective tax t_{ki}. Now there are two possible location choices. 1) The plant locates in country i, and hence faces tax $t_{ki}= t_i$. 2) The plant locates in country j . Under the No-BCA-regime, the plant will simply face the tax imposed by country j, i.e., $t_{ki}= t_j$. Under the BCA-regime, the same is true as long as $t_i \leq t_j$. However, if $t_i > t_j$, then under the BCA-regime, this firm faces the effective tax $t_{ki} = t_j + \theta(t_i - t_j)$, with θ the border tax adjustment parameter (Eyland and Zaccour, 2014).

The simultaneous maximisation of profits obtained in market i by both firms, i.e. π_{1i} is maximised with respect to output x_{1i} and π_{2i} with respect to x_{2i}, gives the quantities supplied by firm 1 and 2 in market i

$$x_{1i} = \frac{A - 2t_{1i} + t_{2i}}{3} \ \&\ x_{2i} = \frac{A - 2t_{2i} + t_{1i}}{3} \ \forall\ i = 1, 2 \tag{3}$$

with $A = a - c$, which we interpret as a market size parameter. Clearly, output levels need to be non-negative. We will test this later for each location equilibrium. Each firm takes separate quantity decisions for the supply to the home and foreign market. Accordingly, profits obtained in market i are given by:

$$\pi_{1i} = (x_{1i})^2 \ \&\ \pi_{2i} = (x_{2i})^2 \ \forall\ i = 1, 2\,. \tag{4}$$

Thus, the final stage of the three stage game is a Nash equilibrium in output levels in each of the two markets. As both firms are assumed to be identical in all respects and, as will be explained later, there are neither fixed nor transportation costs, different profits only stem from differences in effective taxes which firms face.

In the second stage, firms choose their location for each of their two plants simultaneously. That is, they take a decision for each market separately. Generally speaking, firm k supplying market i, compares its profit from locating in country i $\pi_{ki}(i)$ with its profit locating in country j $\pi_{ki}(j)$. As this comparison will generally depend on where the competitor firm ℓ locates, the comparison for market i is based on computing $\Delta\pi_{ki} = \pi_{ki}(i,\ell) - \pi_{ki}(j,\ell)$, $\ell = i, j$, with the first entry in brackets indicating the location of firm k and the second entry the location of the competitor firm ℓ. For a given location of firm ℓ , firm k will locate in country i if $\Delta\pi_{ki} > 0$ and will locate in country j if $\Delta\pi_{ki} < 0$. In case of indifference, $\Delta\pi_{ki} = 0$, we assume that a firm´s plant locates in the country of origin. The equilibrium location choice implies mutual best replies by firm ℓ and k, with respect to their plants supplying market i. That is, the solution of the second stage is a Nash equilibrium of location choices of plants supplying a particular market. As each firm has two plants, each firm takes two location decisions.

In the first stage, governments choose the level of their emission/output tax t_i based on the following welfare function:

$$W_i = CS_i + PS_i + T_i - D_i + BCA_i \tag{5}$$

where CS_i is the consumer surplus in country i, with the consumer surplus being given by $CS_i = \frac{X_i^2}{2}$ which follows from (1), recalling that the total supply to market i is given by $X_i = x_{1i} + x_{2i}$. PS_i is the producer surplus, which is equal to the sum of profits of plants located in country i. T_i is the tax revenue of government i where $T_i = t_i X^i$ and X^i is the sum of output levels produced in country i. D_i are damages from pollution which are released in the production of good x. We assume a global pollutant and hence damages in country i depend on total emissions, E, regardless of the location of the source of emissions. Hence, damages in country i are $D_i(E)$, $E = X = \sum X^i = \sum X_i$. That is, as we normalise the emission-output coefficient to 1, global emissions are equal to total production, which is equal to total consumption. More specifically, we assume:

$$D\left(E\right) = dE, \quad D_1 = \gamma D(E), \quad D_2 = (1-\gamma)D(E), \gamma \in [0.5, 1] \tag{6}$$

with $d > 0$ a damage parameter, reflecting global marginal damages. Hence, country 1 suffers a portion γ and country 2 a portion $(1-\gamma)$ of global damages where we allow for the possibility that countries perceive or evaluate those damages differently. Given $\gamma \in [0.5, 1]$, country 1 is at least as concerned as country 2 about environmental damages and normally more whenever γ is strictly larger than 0.5. Clearly, this implies two benchmarks: a) $\gamma = 0.5$ implies a symmetric damage evaluation in both countries and b) $\gamma = 1$ implies that country 2 is not concerned about environmental damages at all.

Finally, the last term in the welfare function, BCA_i, stands for the tariff revenues from a border carbon adjustment policy. This term is different for our two policy regimes. Under the No-BCA-regime, this term is zero by assumption. Under the BCA-regime, this term is positive for the government which imposes border carbon adjustment but zero for the other government. Generally, $BCA_i = \theta\,(t_i - t_j)\,[x_{ki}(j) + x_{\ell i}(j)]$ if and only if $t_i > t_j$, otherwise $BCA_i = 0$. We assume $\theta = 1$, not only for simplicity but for two other reasons. Firstly, any value of θ above 1 would not be compatible with the equal treatment rule under WTO. Secondly, any value smaller than 1 would not be optimal for country i if it has the option to use BCAs.[6] Normally, and as it will be demonstrated below, in equilibrium, either none, or only one firm will supply market i being located in country j, but generally, it could be both firms supplying market i from abroad (which explains why we include two terms in the squared brackets in the definition of BCA_i above).

2.2 Basic Features of the Model

In this subsection, we discuss the main assumptions of the model with a closer look at the welfare components mentioned above. Firstly, we assume that consumption takes place in the two countries under consideration and hence, consumers matter in our model. This is not only important because a crucial feature of BCAs, which are a form of tariff, is their negative impact on consumers, but, even more fundamentally, without consumers there are no imports on which BCAs could be applied.[7] Secondly, and as mentioned above, BCAs are mainly proposed to internalise global externalities. Hence, it only makes sense to assume global pollution.

Thirdly, we assume that in a government´s welfare function profits of plants located in its country as well as the tax revenues obtained from these plants fully matter. This in line with for instance Ulph and Valentini (2001) and Petrakis and Xepapadeas (2003), who assume that profits and tax revenues go to the country in which production takes place. A wider interpretation is that profits are an indicator of the importance of domestic production and associated jobs. This in line with the main argument put forward by governments and lobby groups for not implementing too strict climate policies.[8] Moreover, excluding profits from a government´s welfare

[6] It is straightforward to show that if country i was to to chose θ endogenously, it would choose a value strictly larger than 1.

[7] Under the No-BCA regime and if production was sold to a third market, the incentive to set lax environmental standards/low taxes would be reduced in our model. See for example Ulph and Valentini (2001).

[8] Some papers do not consider profits in the welfare function of governments or assume full repatriation of profits. For instance, Markusen et al. (1995) assume that profits are distributed

function implies that tax competition between governments with a possible 'race to the bottom' would be mainly driven by tax revenue considerations by governments, which is also questioned by Rauscher (1995) as not being very convincing. Furthermore, without considering profits, the race to the bottom phenomenon could not be explained for other types of environmental regulations, like environmental standards, which do not generate revenues to governments.

Fourthly, we assume that location choices of firms depend only on tax differentials, i.e., we abstract from other location-specific costs that could affect profits of firms like transportation and set-up costs. We do not consider these costs, not only because of their obvious effects, but also to focus our analysis on the role of BCAs for firms´ location decisions and the tax competition game between governments.[9] More importantly, as convincingly argued by Hoel (1997), including those costs do not alter qualitative results, but adds greatly to the complexity of models.[10]

Regarding the revenues from border carbon adjustments, the term BCA_i only appears under the BCA-regime and only in the welfare function of the country which imposes a higher carbon tax. Given our assumption that BCAs fully adjusts the difference between the two taxes (i.e., the effective tax of firm k being located in country j and supplying market i, $t_{ki} = t_j + \theta(t_i - t_j)$, is simply $t_{ki} = t_i$ for $\theta = 1$), the BCA-regime can make a difference to the home market i as not only the home firm´s but also the foreign firm´s supply to the home market face de facto the same tax t_i provided $t_i > t_j$. This implies that all plants supplying country i are subject to the same carbon tax irrespective of the location of production. In other words, BCAs partially "protect" the home firm´s profit by creating an equal playing field in market i. It is only partial and not full protection because in the foreign market j, the foreign firm will have a competitive advantage over the home firm provided $t_i > t_j$. Of course, the home firm can avoid this problem by relocating its plant to the foreign country j for the supply of this market (and hence facing t_j instead of t_i for its supply to market j).

As it will become evident, modeling plant location as an endogenous choice of firms, poses some analytical difficulties. It causes not only discontinuous location choices of firms as a function of taxes, but, more importantly, may also causes best response functions of governments to be discontinuous, which, in our model, may lead to the

throughout the world, whereas Rauscher (1995) considers that profits accrue to a foreign investor. Both Motta and Thisse (1994) and Eerola (2006) assume that profits remain in the country of the headquarter of a company. Clearly, excluding profits from a government´s welfare function would weaken the impacts of the relocation of firms for governments, and hence tax competition among countries becomes less fierce. See Janeba (1998) and Ulph and Valentini (2001) on this point.

[9]Fixed costs or set-up costs of plants reduce the incentive of plant relocation. In contrast, transportation costs increases the incentive of relocation for the plant that supplies the foreign market.

[10]For instance, the model by Markusen et al. (1995) considers transportation and fixed costs and hence relies on numerical simulations. In contrast, Rauscher (1995) and Hoel (1997) abstract from those costs, obtain similar results, though analytically.

non-existence of a Nash equilibrium under the BCA-regime. It is for this reason that apart from Nash equilibria we also determine Stackelberg equilibria in this first stage of our three stage game. Moreover, for expositional simplicity, we consider only the possibility of a unilateral BCA-policy which is imposed by country 1, which is more concerned about environmental damages in our model. The general possibility of a bilateral BCA-policy where country 2 could also impose a tariff if $t_i < t_j$ is considered in Appendix D where it is shown that all qualitative conclusions continue to hold.

2.3 Location Equilibria

Firms choose the location of their plants in the second stage based on the carbon taxes chosen by governments in the first stage. Given that we abstract from transportation and set-up costs, the decision of firms only depends on tax differentials as demonstrated in more detail in Appendix A.1 Under the No-BCA-regime, only the location of production matters, which gives rise to three location equilibria. 1) 'No relocation' (NR) if $t_1 = t_2$. Each firm remains with its two plants in the country of origin. 2) Total relocation of firm 1 (TR_1) if $t_1 > t_2$. That is, firm 1, originally located in country 1, will relocate with both plants to country 2. 3) Total relocation of firm 2 (TR_2) if $t_1 < t_2$. That is, firm 2, originally located in country 1, will relocate with both plants to country 2. Under the BCA-regime, assuming that only government 1 can impose BCAs, location equilibria NR if $t_1 = t_2$ and TR_2 if $t_1 < t_2$ are the same. However, if $t_1 > t_2$, the TR_1-location equilibrium disappears in favour of the PR_1-location equilibrium, standing for 'partial relocation of firm 1'. Firm 1´s plant supplying market 2 will relocate to country 2, but its plant supplying its own market in country 1 will remain in the country of origin as also the foreign firm 2 faces de facto the same tax t_1 on its exports to the market in country 1 (and, as pointed out above, we assume that in case of indifference plants do not relocate). That is, country 1 imposing BCAs on imports can avoid total relocation of its firm 1, but cannot avoid partial relocation. If we considered that also country 2 can impose BCAs on imports if $t_1 < t_2$, as we do in Appendix D, then also location equilibrium PR_2 would exist.

2.4 Normative Benchmark

Before turning to non-cooperative equilibria under the two policy regimes, we briefly discuss the normative benchmark of the social optimum. Maximising $W_1 + W_2$ with respect to output levels, delivers X_1^C and X_2^C, the socially optimal output levels supplied to market 1 and 2, with $X_1^C = x_{11} + x_{21}$ and $X_2^C = x_{12} + x_{22}$. The composition of X_1^C and X_2^C does not matter as we assume linear and identical production costs for all plants. Moreover, due to a global pollutant (and hence only aggregate damages matter in the social optimum) and because of symmetric consumers, $X_1^C = X_2^C$ must be true. In order to determine the the socially optimal tax, we need to take two steps.

Firstly, considering BCAs in country i with $t_i > t_j$ and using equilibrium output levels (3), gives $X_i = 2(A - t_i)/3$ for market i and $X_j = 2(A - t_j)/3$ for market j from which it is evident that $X_i^C = X_j^C$ is impossible. Secondly, without BCAs, $X_1^C = X_2^C$ is possible under three location equilibria: a) NR with $t_1 = t_2 = t_C$. b) TR_1 with $t_1 > t_2 = t_C$ and c) TR_2 with $t_C = t_1 < t_2$. These three location equilibria de facto imply the same effective tax rate t_C imposed on all firms.

Proposition 1. *Social Optimum*

The socially optimal output levels are given by $X_1^C = X_2^C = A - d$. Under the No-BCA-regime, the effective socially optimal tax is given by

$$t_C = -\frac{1}{2}A + \frac{3}{2}d \ . \tag{7}$$

with associate output levels $x_{11}^C = x_{12}^C = x_{21}^C = x_{22}^C = (A - d)/2$. A BCA-regime cannot generate socially optimal output levels and hence global welfare will be strictly lower.

Hence, the effective socially optimal tax rate is unique, is associated with a unique output vector if generated by a tax, though it is associated with three different possible location equilibria and tax vectors. Hence, in the following, it is helpful to think of the socially optimal tax as a uniform tax imposed in both countries for simplicity. It is interesting to note that - as we will demonstrate later - BCAs can increase global welfare in a non-cooperative equilibrium, but as Proposition 1 states, they will always fall short of achieving the socially optimal global welfare level. Finally, note that output levels need to be non-negative and since we are only interested in interior solutions, we assume henceforth $A > d$. It is also evident from the equilibrium output levels above that a necessary condition to ensure positive production levels requires $A > t_i$, $i \in \{1, 2\}$, of which we make use in the subsequent analysis.

3 Climate Policy Equilibria: No-BCA-Regime

In this section, we solve the first stage of our game under the No-BCA-regime. Based on subsection 2.3, the three possible location equilibria under this regime are: 1) NR if $t_1 = t_2$. 2) TR_1 if $t_1 > t_2$, i.e. all plants are located in country 2. 3) TR_2 if $t_1 < t_2$, i.e. all plants are located in country 1.

Countries choose their climate policies in two games: a simultaneous game and a sequential game associated with the notions of Nash equilibrium (NE) and Stackelberg equilibrium (SE), respectively. To solve both games, we need to derive the reaction function of each country. This proceed through three steps. *Firstly*, we write the welfare functions of each country under the three possible location equilibria. *Secondly*, we analyse the best response of each country given a location equilibrium. *Thirdly*, we analyse the best response of each country across the possible location equilibria.

From (5), the welfare function of each country depends on the location equilibrium as follows:

$$W_i = \begin{cases} W_i^{TR_\ell} = CS_i + \sum_{k=1,2} \pi_{k1} + \sum_{k=1,2} \pi_{k2} + T_i - D_i & \text{if } t_i < t_j \quad (8a) \\ W_i^{NR} = CS_i + \pi_{k1} + \pi_{k2} + T_i - D_i & \text{if } t_i = t_j \quad (8b) \\ W_i^{TR_k} = CS_i - D_i & \text{if } t_i > t_j \quad (8c) \end{cases}$$

where the superscript TR_ℓ refers to the total relocation of the foreign firm ℓ from country j, and TR_k refers to the total relocation of the home firm k.

In the following, we analyse the best response of each country given a location equilibrium. Inserting the equilibrium output levels provided in Appendix A.2 into (8a), (8b), and (8c) gives (9),(11) and (12), respectively. We consider three cases in turn.

First: given $t_i < t_j$, the two firms are located in country i and its welfare level is given by:

$$W_i^{TR_\ell} = \frac{2}{3}(A - t_i)\left(A + t_i - 2D_i^{'}\right) \tag{9}$$

where $D_i^{'}$ is the individual marginal damage, i.e. $D_1^{'} = \gamma d$ and $D_2^{'} = (1-\gamma)d$.

The first-order condition is given by:

$$\frac{\partial W_i^{TR_\ell}}{\partial t_i} = \frac{4}{3}\left(D_i^{'} - t_i\right) = 0 \Leftrightarrow \hat{t}_1^{TR_2} = \gamma d \text{ and } \hat{t}_2^{TR_1} = (1-\gamma)d \tag{10}$$

The optimal carbon tax in this location equilibrium is equal to the country's marginal damage. On the one hand, country i has an incentive to subsidise its consumers to correct market distortions resulted from imperfect competition (Barnett, 1980). On the other hand, and different from the standard profit-shifting argument, the government has an incentive to tax producers.[11] Since profits of both firms contribute to the welfare function, the government could maximise profits of firms by taxing producers to enforce a monopolist output. In addition, the incentive to internalise environmental damages calls for a tax. We find that incentives to subsidise consumers cancel out incentives to tax producers, hence the optimal carbon tax reflects only the environmental motive. Therefore, if $t_j > D_i^{'}$, the best response of country i

[11] Under the profit-shifting argument, a government has an incentive to subsidise its domestic producer at the expense of the rival firm (Brander and Spencer, 1985). However, this is true if firms, hence their profits, are located in different countries.

is to impose its unconstrained carbon tax, which is equal to its marginal damages. However, if $t_j \leq D_i^{'}$, country i deviates from its optimal tax, where its best response would be to undercut country j 's tax which is considered the highest possible tax in this case.

Second: given $t_i = t_j$, the welfare level under NR is written as:

$$W_i^{NR} = \frac{4}{9}\left(A - t_i\right)\left(A + \frac{1}{2}t_i - 3D_i^{'}\right) \tag{11}$$

Obviously, for any given tax t_j, country i cannot do better than setting the same tax level to keep its firm, i.e. matching taxes is always the best response in this location equilibrium.

Third: given $t_i > t_j$, all plants are located in country j. In this case, the welfare of country i is given by the consumer surplus minus environmental damages, where both are function of t_j as follows:

$$W_i^{TR_k} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_j\right)^2 - D_i^{'}\left(\frac{4}{3}\left(A - t_j\right)\right) \tag{12}$$

From (12), t_j has two opposing effects on country i in this location equilibrium. The consumer surplus is decreasing in t_j, while reduction of damages is increasing in t_j. The welfare level $W_i^{TR_k}$ is convex in t_j, where it reaches its minimum at $t_j = A - 3D_i^{'}$.

To sum up, we need to consider three cases for each country. We start with country 1, for which the three cases are listed below as follows:

1. In the case $t_1 < t_2$:
   (a) Let $\hat{W}_1^{TR_2}$ be $W_1^{TR_2}\left(t_1 = \hat{t}_1^{TR_2}\right)$ in the range $t_2 > \hat{t}_1^{TR_2} = \gamma d$, i.e. country 1 sets its optimal tax when it has both firms and t_2 is sufficiently high.
   (b) Let $\tilde{W}_1^{TR_2}$ be $W_1^{TR_2}\left(t_1 = t_2 - \varepsilon\right)$, as a function of t_2, in the range $t_2 \leq \hat{t}_1^{TR_2} = \gamma d$, i.e. country 1 cannot set its optimal tax and deviates by undercutting the tax of country 2 to have both firms, where $\varepsilon > 0$ and ε is arbitrarily small and close to zero.
2. In the case $t_1 = t_2$: let W_1^{NR} be $W_1^{NR}\left(t_1 = t_2\right)$, as a function of t_2, where each firm remains in its home country.
3. In the case $t_1 > t_2$: let $W_1^{TR_1}$ be $W_1^{TR_1}\left(t_1 > t_2\right)$, as a function of t_2, where both firms are located in country 2.

Substituting the tax levels defined above into (9), (11) and (12), we obtain the welfare function of country 1 under each case. Details are provided in Appendix A.2.

Finally, we rank the above welfare functions for different levels of t_2. We start in Fig. 1 with a range, where $t_2 > \gamma d$. Obviously in this range, country 1 achieves the highest welfare level under TR_2, where it attracts both firms and can freely control its damages by imposing its unconstrained carbon tax, i.e. γd. Hence, $\hat{W}_1^{TR_2}$ is a constant function throughout this range. From (8b) and (8c) the difference between the welfare levels under NR and TR_1 is profits plus taxes, which is positive in this range.

However, if country 2 sets $t_2 \leq \gamma d$, country 1 responds by its constrained carbon tax for this range, i.e. by undercutting t_2, if it would like to attract both firms. Hence, its welfare level under TR_2, i.e. $\tilde{W}_1^{TR_2}$, starts to decrease. Looking at the three curves in this range, it is clearly shown that as t_2 decreases, the gap between the three curves becomes smaller. The intuition here is that a change in t_2 at the margin will have a minor effect on both the consumer surplus and environmental damages, which always contribute to the welfare level irrespective of the location equilibrium. On the other hand, a change in t_2 will have a discrete effect on both profits and tax revenues, which accrue only to the country in which production takes place. As long as $-\frac{1}{2}A < t_2$, profits plus taxes (minus subsidies) is positive, hence undercutting to attract both firms pays. However, undercutting stops and the three curves converge at $t_2 = -\frac{1}{2}A$, where profits minus subsidies becomes zero. Further undercutting, i.e. if $t_2 < -\frac{1}{2}A$, means too much subsidy for both firms, hence it is not a best response.

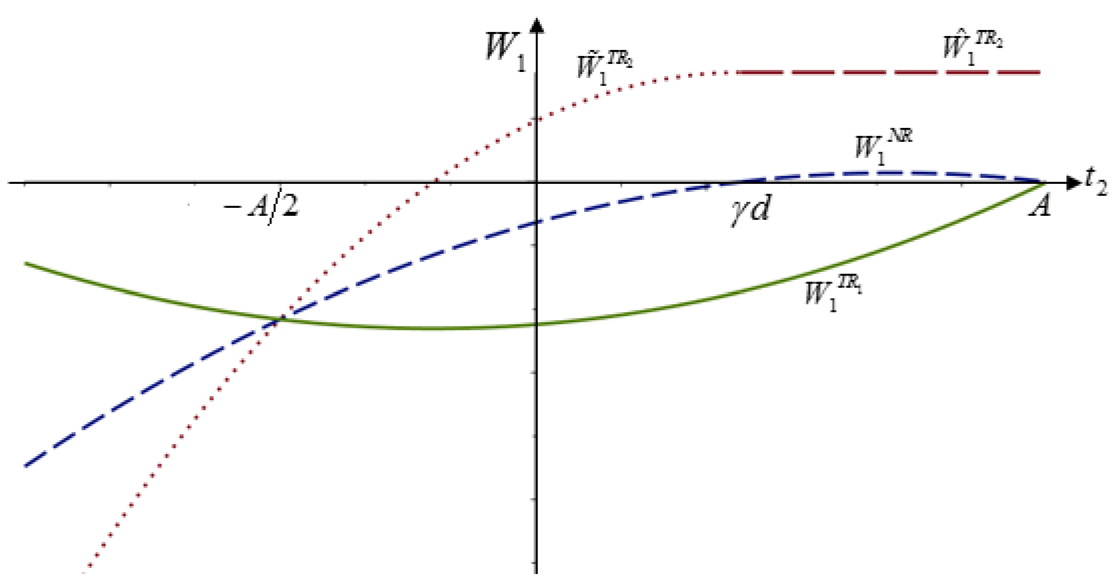

Fig. 1: Ranking of the Welfare Levels of Country 1 without BCAs

Similarly, we consider the following three cases for country 2:

1. In the case $t_1 > t_2$:

(a) Let $\hat{W}_2^{TR_1}$ be $W_2^{TR_1}\left(t_2 = \hat{t}_2^{TR_1}\right)$ in the range $t_1 > \hat{t}_2^{TR_1} = (1-\gamma)\,d$, i.e. country 2 sets its optimal tax when it has both firms and t_1 is sufficiently high.

(b) Let $\tilde{W}_2^{TR_1}$ be $W_2^{TR_1}\left(t_2 = t_1 - \varepsilon\right)$ as a function of t_1, in the range $t_1 \leq \hat{t}_2^{TR_2} = (1-\gamma)\,d$ i.e. country 2 cannot set its optimal tax and deviates by undercutting the tax of country 1 to have both firms, where $\varepsilon > 0$ and ε is arbitrarily small and close to zero.

2. In the case $t_2 = t_1$: let W_2^{NR} be $W_2^{NR}\left(t_2 = t_1\right)$ as a function of t_1, where each firm remains in its home country.

3. In the case $t_1 < t_2$: let $W_2^{TR_2}$ be $W_2^{TR_2}\left(t_1 < t_2\right)$ as a function of t_1, where both firms are in country 1.

Inserting the tax levels defined above into (9), (11) and (12), delivers the welfare function of country 2 under each case. See Appendix A.3 for the details. An analogues analysis holds for country 2. Hence, the best response of countries is summarised in the following Lemma.

Lemma 1. The Best Response of Countries under the No-BCA-Regime

i. If $D_i' < t_j < A$, the welfare levels can be ranked as follows: $\hat{W}_i^{TR_\ell} > W_i^{NR} > W_i^{TR_k}$.

ii. If $-\frac{1}{2}A < t_j \leq D_i'$, the welfare levels be ranked as follows: $\tilde{W}_i^{TR_\ell} > W_i^{NR} > W_i^{TR_k}$.

iii. If $t_j \leq -\frac{1}{2}A$ the welfare levels can be ranked as follows: $W_i^{TR_k} \geq W_i^{NR} \geq \tilde{W}_i^{TR_\ell}$.

Hence, the best response of country 1 and country 2 are given respectively by:

$$t_1 \begin{cases} = \gamma d & \text{if } \gamma d < t_2 < A \\ = t_2\text{-}\varepsilon & \text{if } -\frac{1}{2}A < t_2 \leq \gamma d \\ \geq t_2 & \text{if } t_2 \leq -\frac{1}{2}A \end{cases} \tag{13}$$

and

$$t_2 \begin{cases} = (1-\gamma)\,d & \text{if } (1-\gamma)\,d < t_1 < A \\ = t_1\text{-}\varepsilon & \text{if } -\frac{1}{2}A < t_1 \leq (1-\gamma)\,d \\ \geq t_1 & \text{if } t_1 \leq -\frac{1}{2}A \end{cases} \tag{14}$$

Proof. See Appendix A.4. □

The reaction function illustrated in Fig. 2 has three parts. The first is a vertical (horizontal) line referring to the dominant strategy. That is, each country sets its unconstrained carbon tax, i.e. D_i'. The second part lies slightly above (below) the 45° line and is upward sloping, which implies that carbon taxes are strategic complements. In this part, the best response of countries is to undercut each other's tax i.e. they choose a constrained carbon tax. Although the third part is not drawn, it refers to any tax level larger than the subsidy level $-\frac{1}{2}A$. These reaction functions are different from the downward sloping reaction functions under models which assume fixed plant location, where carbon taxes are strategic substitutes. It will be demonstrated below that competition becomes more fierce when the location of firms is endogenous.

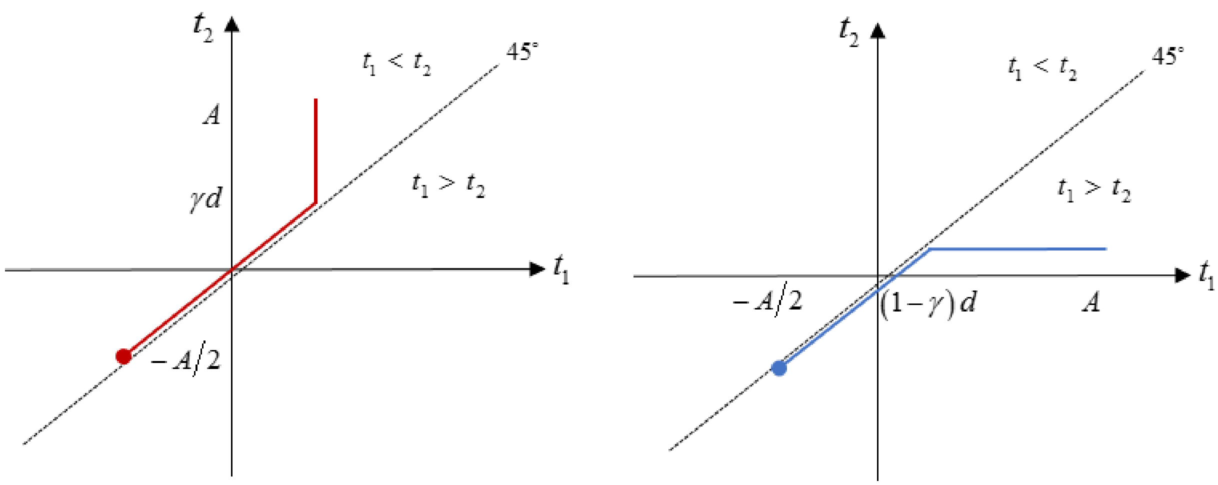

Fig. $(2a)$: Country 1 Fig. $(2b)$: Country 2

Fig. 2: Reaction Function of Countries without BCAs

3.1 Simultaneous Game

In this subsection, countries solve the first stage simultaneously. In Fig. 3 below, we combine the reaction functions of both countries, from which we obtain the unique pure strategy NE without BCAs. We denote by $\left(t_1^{*NE}, t_2^{*NE}\right)$ the Nash equilibrium of this game.

Proposition 2. Nash Equilibrium under the No-BCA-Regime

Without BCAs, if countries choose their climate policy simultaneously, the unique pure strategy Nash equilibrium is a subsidy: $t_1^{*NE} = t_2^{*NE} = -\frac{1}{2}A$*, which is a 'race to the bottom', and the location equilibrium of firms is* NR*.*

Proof. Follows directly from Fig. 3. □

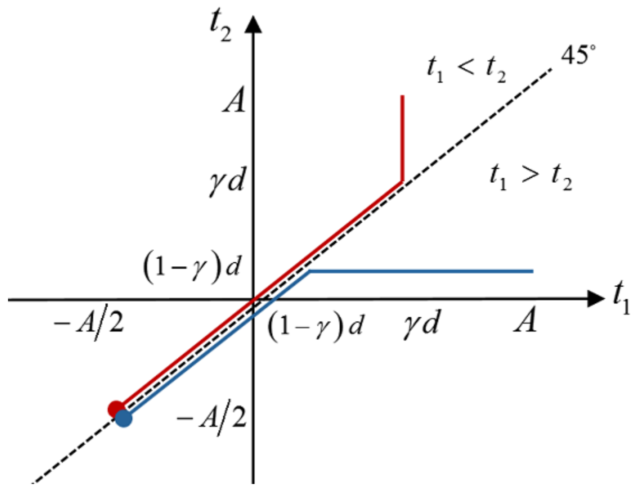

Fig. 3: Nash Equilibrium without BCAs

In this equilibrium, both countries are stuck in the bottom, where the fear of losing firms discourages countries from choosing higher carbon taxes, a phenomenon known as 'regulatory chill' (Neumayer, 2001). Although governments have incentives to weaken their environmental policies if the location of firms is either fixed or endogenous, a 'race to the bottom' occurs only in an endogenous plant location model due to the strategic complementarity of carbon taxes. If the location of firms is fixed, governments set lower carbon taxes to shift profits from the foreign to the home firm. However, under endogenous location models, this incentive is reinforced to prevent the total loss of profits if the home firm relocates to the foreign country.

Proposition 2 highlights two observations. First, we obtain a symmetric NE irrespective of the asymmetry among countries in terms of their evaluation of damages, a result which never arise under models of fixed plant location. Even though country 1 perceives damages from emissions more importantly than country 2, it is rational to set lower taxes in this model if country 2 does so, to avoid losing firms.This result emerges because we consider a global pollution. If we assume local pollution instead, we may have an asymmetric NE equilibrium.[12] Hence combining both asymmetry and global pollution leads to a symmetric NE under endogenous plant location. Second, unlike models that assume local pollution, we have a unique NE. As explained above, both countries in our model suffer from global damages irrespective of the location of production, i.e. emissions. Hence, governments undercut each other's carbon tax until gains from both profits and tax revenues are exhausted. Our result is similar to Rauscher (1995) who also considers a case of a global pollution, under which countries compete towards a zero tax rate.[13] While under local

[12] If we assume local pollution, we may find asymmetric NE, with all plants located in country 2. See also Rauscher (1995) and (Hoel, 1997).

[13] In our model, there is a competition towards a subsidy level. The difference is that he does not consider profits in the welfare function of countries, and he assumes monopoly not oligopoly as in

pollution, countries suffer from damages generated from local production only. This might lead to a second equilibrium with sufficiently high carbon tax such that firms exit the market, i,e, NIBY equilibrium (Markusen et al., 1995).

3.2 Sequential Game

In this subsection, countries choose their climate policies in the first stage sequentially. We consider the two orders of moves in turn.

Firstly, if country 1 leads, it takes into account the best response of the follower which is given in (14). As a leader, country 1 knows that it cannot attract firm 2 for all $t_1 > -\frac{1}{2}A$, while for all $t_1 < -\frac{1}{2}A$, it has no incentive to attract both firms. Therefore, to keep its home firm, country 1 will set the tax level at which country 2 will no longer be willing to undercut to attract firm 1, i.e. $t_1 = -\frac{1}{2}A$. In this case, country 2 is indifferent between setting the same tax, i.e. NR, or a higher tax, i.e. TR_2. Country 1 is also indifferent between the two outcomes.

Let t_i^L denotes the tax of the leader, while t_i^F denotes the tax of the follower. If country 1 sets its tax level at $t_1^L = -\frac{1}{2}A$, there are basically two Stackelberg equilibria: $t_1^L = t_2^F = -\frac{1}{2}A$, i.e. NR, and $t_1^L = -\frac{1}{2}A < t_2^F$, i.e. TR_2. Both countries are indifferent in these two equilibria. Therefore, we need a tie-breaking rule to have a unique equilibrium at this tax level. We assume that if both countries are indifferent, the follower chooses no relocation, i.e. the Stackelberg equilibrium (SE) is $t_1^L = t_2^F = -\frac{1}{2}A$. This assumption will not affect the results since each country has the same welfare level under both equilibria. However, it makes the SE easily comparable to the NE.

Alternatively, country 1 might impose a tax large enough to induce firms to locate in country 2, i.e. $t_1 > t_2$. In this case, we find that country 1 will choose a tax high enough such that country 2 responds by setting its optimal tax rate, i.e. $\hat{t}_2^{TR_1}$.[14]

Facing these two options, country 1 will choose to let its firm to relocate if:

$$W_1^{*TR_1}\left(t_1^{*L} > t_2^{*F} = (1-\gamma)\, d\right) > W_1^{*NR}\left(t_1^{*L} = t_2^{*F} = -\frac{1}{2}A\right) \text{ if } A < \hat{A}_1^{TR_1}$$

where $\hat{A}_1^{TR_1} = 2\gamma d + \frac{2}{5}d$. Hence, the following proposition shows two possible Stackelberg equilibria, which depend on the parameter range.

Proposition 3. Stackelberg Equilibrium under the No-BCA-Regime if Country 1 Leads

i. Equilibrium 1 is a subsidy where, $t_1^{*L} = t_2^{*F} = -\frac{1}{2}A$*, and the equilibrium location of firms is* NR *if* $A > \hat{A}_1^{TR_1}$.

our model.

[14] See the proof of Proposition 3 below.

ii. Equilibrium 2 is a tax where, $t_1^{*L} > t_2^{*F} = (1-\gamma)\, d$, *and the equilibrium location of firms is* TR_1 *if* $A \leq \hat{A}_1^{TR_1}$.[15]

iii. $\hat{A}_1^{TR_1}$ *increases in* γ *and* d .

Proof. See Appendix A.5. □

The first SE is exactly the same as the NE, i.e. a 'race to the bottom'. However, country 1 could avoid being stuck in the bottom and choose the second SE if $A < \hat{A}_1^{TR_1}$. In the two equilibria stated above, profits plus taxes (minus subsidies) is zero in country 1. Hence, whether country 1 chooses to let its firm to relocate depends on the trade-off between its environmental damages and consumer surplus. Intuitively, the larger γ and d , the larger marginal damages of country 1, and the larger the threshold level under which country 1 chooses equilibrium 2. In this case, gains from reduction in environmental damages under TR_1 outweigh the loss of consumer surplus compared to the NR equilibrium.

Secondly, if country 2 leads, it takes into account the reaction function of country 1 in (13). Clearly, also country 2 is not able to attract any plant of firm 1 for all $t_2 > -\frac{1}{2}A$. In addition, it has no incentive to subsidise both firms for $t_2 < -\frac{1}{2}A$. Therefore, country 2 has the same choices as country 1. We find that country 2 prefers to let its firms go if:

$$W_2^{*TR_2}\left(t_2^{*L} > t_1^{*F} = \gamma d\right) > W_2^{*NR}\left(t_2^{*L} = t_1^{*F} = -\frac{1}{2}A\right) \text{ if } A < \hat{A}_2^{TR_2}$$

where $\hat{A}_2^{TR_2} = 2(1-\gamma)d + \frac{2}{5}d$.

Proposition 4. Stackelberg Equilibrium under the No-BCA-Regime if Country 2 Leads

i. Equilibrium 1 is a subsidy, $t_2^{*L} = t_1^{*F} = -\frac{1}{2}A$, *and the equilibrium location of firms is* NR *for all* $\gamma \geq 0.7$ *or for all* $\gamma < 0.7$ *if* $A > \hat{A}_2^{TR_2}$.

ii. Equilibrium 2 is a tax, $t_2^{*L} > t_1^{*F} = \gamma d$, *and the equilibrium location of firms is* TR_2 *for all* $\gamma < 0.7$ *if* $A \leq \hat{A}_2^{TR_2}$.

iii. $\hat{A}_2^{TR_2}$ *decreases in* γ *and increases in* d.

Proof. See Appendix A.5. □

[15] If $A = \hat{A}_1^{TR_1}$, country 1 is indifferent between the two equilibria. In this case, we select the SE according to a Pareto dominance criteria, i.e. the equilibrium which gives the follower a higher welfare level.

The larger the marginal damages of country 2, i.e. the smaller γ, the more likely it chooses to let its firm to relocate. However, as γ increases, country 2 suffers less damage from global emissions, thus it is less likely that TR_2 is attractive. That is, if countries are highly asymmetric, i.e. $\gamma \geq 0.7$, TR_2 never emerge as a location equilibrium for any given value of A and d. In such cases, a larger consumer surplus under NR always outweighs less damages that could be achieved under TR_2.

3.3 Comparisons under the No-BCA-Regime

It is worthwhile at this point to compare the outcomes of the two policy games, and to show how non-cooperative equilibria can be compared to the social optimum.

Corollary 1. *Without BCAs, i. a sequential instead of a simultaneous choice of climate policies is Pareto-improving for both countries and leads to less global emissions if marginal damages of the Stackelberg leader are sufficiently large.*

ii. The non-cooperative carbon policies in both the simultaneous and the sequential game are less strict than the socially optimal carbon tax.

Proof. See Appendix A.6. □

Moving sequentially could allow the Stackelberg leader to avoid the 'race to the bottom' by letting its home firm to relocate. In such cases, the follower attracts all plants, which face a higher carbon tax than under the NE. Hence, global emissions are reduced. Moreover, this SE is Pareto-improving for both countries, hence resulting in a higher global welfare level. We find that relocation of firms is more likely to occur under the leadership of country 1, which is environmentally more concerned, i.e. $\hat{A}_1^{TR_1} > \hat{A}_2^{TR_2}$. However, even though the SE could improve upon the NE, the socially optimum outcome is not obtained under the two policy games in this regime.

4 Climate Policy Equilibria: BCA-Regime

As mentioned in section 2, we consider a situation, under which the BCA-policy is imposed unilaterally by county 1, i.e. no BCAs in country 2 if $t_2 > t_1$.[16] Hence, everything will be the same as in the previous section, except for the case $t_1 > t_2$. Therefore, we consider the following three possible location equilibria under this regime: 1) NR if $t_1 = t_2$. 2) PR_1 if $t_1 > t_2$, i.e. firm 1 relocates partially with its plant that supplies country 2 only. 3) TR_2 if $t_1 < t_2$, i.e. all plants are located in country 1.

[16] See Appendix D for the case if both countries impose BCAs.

Under this regime, countries have different incentives, for this reason we treat each country separately in this section. We follow the same steps as before. Thus, we firstly list below the welfare functions of country 1 and country 2, respectively, under the three possible location equilibria.

$$W_1 = \begin{cases} W_1^{TR_2} = CS_1 + \pi_{11} + \pi_{12} + \pi_{21} + \pi_{22} + T_1 - D_1 & \text{if } t_1 < t_2 \quad (15a) \\ W_1^{NR} = CS_1 + \pi_{11} + \pi_{12} + T_1 - D_1 & \text{if } t_1 = t_2 \quad (15b) \\ W_1^{PR_1} = CS_1 + \pi_{11} + T_1 + BCA_1 - D_1 & \text{if } t_1 > t_2 \quad (15c) \end{cases}$$

where $BCA_1 = (t_1 - t_2)\, x_{21}$.

$$W_2 = \begin{cases} W_2^{TR_2} = CS_2 - D_2 & \text{if } t_1 < t_2 \quad (16a) \\ W_2^{NR} = CS_2 + \pi_{21} + \pi_{22} + T_2 - D_2 & \text{if } t_1 = t_2 \quad (16b) \\ W_2^{PR_1} = CS_2 + \pi_{12} + \pi_{21} + \pi_{22} + T_2 - D_2 & \text{if } t_1 > t_2 \quad (16c) \end{cases}$$

Secondly, we analyse the best response of each country given a location equilibrium. In the following, we consider only the case where $t_1 > t_2$, since the other location equilibria are the same as in section 3.

Given $t_1 > t_2$, unlike the previous regime, the BCA-policy allows country 1 to protect one plant of its firm, which serves the home market. Besides the profits and tax revenues of this home plant, country 1 also collects BCAs revenues from imports of the foreign plant. Substituting the equilibrium output levels given in Appendix B.1 into (15c), we obtain:

$$W_1^{PR_1} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_1\right)^2 + \frac{(A-t_1)^2}{9} + t_1\left(\frac{1}{3}(A-t_1)\right)$$
$$(t_1 - t_2)\left(\frac{1}{3}(A-t_1)\right) - \gamma d\left(\frac{4A - 2t_1 - 2t_2}{3}\right) \quad (17)$$

The welfare-maximising tax rate of country 1 under this location equilibrium is given by:[17]

$$\frac{\partial W_1^{PR_1}}{\partial t_1} = \frac{1}{3}(-2t_1 + t_2 + 2\gamma d) = 0 \Leftrightarrow \hat{t}_1^{PR_1}(t_2) = \gamma d + \frac{1}{2}t_2 \quad (18)$$

We call (18) the standard reaction function of country 1 under PR_1, which is a function of t_2 and is upward sloping. That is, for country 1 taxes are strategic complements in this location equilibrium. Since BCAs revenues depend on the difference

[17]The welfare function is strictly concave, $\frac{\partial^2 W_1^{PR_1}}{\partial t_1^2} = -\frac{2}{3} < 0$.

between the two national tax levels, country 1 has an incentive to impose a higher tax if t_2 increases to capture larger tariff revenues.

We need to consider two constraints such that country 1 could set $\hat{t}_1^{PR_1}$ in this location equilibrium. The first is a non-negativity constraint (NNC) that requires $\hat{t}_1^{PR_1} < A$, which ensures positive production levels as mentioned in section 2. The second is a BCA constraint, which requires $t_2 < \hat{t}_1^{PR_1}$. Therefore, we need to analyse the best response of country 1 if any of these two constraints is violated. If the NNC is violated, country 1 deviates from its optimal tax, where its best response would be to set the maximum feasible tax rate, which is $\ddot{t}_1 = A - \varepsilon$. If the BCA constraint is violated, country 1 would respond by overshooting t_2 marginally, i.e. $\breve{t}_1 = t_2 + \varepsilon$, which is the lowest possible tax level higher than t_2 in this case.

Regarding country 2, it would be able to attract one plant only of firm 1 under this regime, in addition to the plants of its home firm if $t_1 > t_2$. Substituting the equilibrium output levels in Appendix B.1 into (16c), the welfare level of country 2 is given by:

$$W_2^{PR_1} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_2\right)^2 + 2\left(\frac{(A-t_2)^2}{9}\right) + \frac{(A-t_1)^2}{9} + t_2\left(A - \frac{2}{3}t_2 - \frac{1}{3}t_1\right) - (1-\gamma)\,d\left(\frac{4A - 2t_1 - 2t_2}{3}\right) \tag{19}$$

The first-order condition from which country 2's optimal tax is derived is given by:[18]

$$\frac{\partial W_2^{PR_1}}{\partial t_2} = \frac{A}{9} - \frac{4}{9}t_2 - \frac{1}{3}t_1 + \frac{2}{3}(1-\gamma)d = 0$$
$$\Leftrightarrow \hat{t}_2^{PR_1}(t_1) = \frac{1}{4}A - \frac{3}{4}t_1 + \frac{3}{2}d\,(1-\gamma) \tag{20}$$

The standard reaction function of country 2 under PR_1, which is derived in (20), is a function of t_1 and is downward sloping. Therefore, from country 2's point of view, carbon taxes are strategic substitutes. This can be explained as follows. If t_1 increases, the plant of firm 2, which supplies country 1, faces a higher carbon tariff which reduces the producer surplus and tax revenues in country 2. Therefore, it has an incentive to raise its tax level to compensate the loss in tax revenues captured by country 1 through BCAs. However, this comes at the expense of consumers and profits of other plants. As a result, the best response of country 2 is to protect consumers and producers through a lower tax level.

[18] The second order condition is satisfied, $\frac{\partial^2 W_2^{PR_1}}{\partial t_2^2} = -\frac{4}{9} < 0$

We also need to consider the best response of country 2 if it would not be able to choose its optimal tax in (20). The BCA constraint requires $t_1 > \hat{t}_2^{PR_1}(t_1)$. For this to be true, we mush have $t_1 > \bar{t}_1$, where $\bar{t}_1 = \frac{1}{7}A + \frac{6}{7}(1-\gamma)\,d$. We find that if the BCA constraint is satisfied, this ensures satisfaction of the NNC, i.e. $\hat{t}_2^{PR_1} < A$. Therefore, if $t_1 > \bar{t}_1$, country 2's best response is to set $\hat{t}_2^{PR_1}$. However, if $t_1 < \bar{t}_1$, country 2 will deviate from its optimal tax and will undercut t_1.

Thirdly, we compare the welfare functions of each country across the three location equilibria to derive the reaction functions under this regime. We start with listing the welfare functions of country 1, for which the third case is the only new compared to the previous regime.

1. In the case $t_1 < t_2$, we have $\hat{W}_1^{TR_2}$ and $\tilde{W}_1^{TR_2}$ as defined in the No-BCA-regime.
2. In the case $t_1 = t_2$, we have W_1^{NR}, which is also defined in the No-BCA-regime.
3. In the case $t_1 > t_2$:
   (a) Let $\hat{W}_1^{PR_1}$ be $W_1^{PR_1}\left(t_1 = \hat{t}_1^{PR_1}(t_2)\right)$ as a function of t_2, in the range $t_2 < 2\gamma d$ and $t_2 < 2A - 2\gamma d$, i.e. country 1 sets the tax level in (18).
   (b) Let $\ddot{W}_1^{PR_1}$ be $W_1^{PR_1}\left(t_1 = \ddot{t_1} = A - \varepsilon\right)$ as a function of t_2, in the range $t_2 > 2A - 2\gamma d$, i.e. country 1 is constrained to set the maximum feasible tax level.
   (c) Let $\breve{W}_1^{PR_1}$ be $W_1^{PR_1}\left(t_1 = \breve{t_1} = t_2 + \varepsilon\right)$ as a function of t_2, in the range $t_2 > 2\gamma d$, i.e. country 1 is constrained to set the lowest possible tax rate, which is higher than t_2.

The details of the welfare functions defined above are given in Appendix B.1

The first range of t_2 in Fig. 4, i.e. $t_2 > \gamma d$, is similar to Fig. 1 under the No-BCA-regime. That is, even with BCAs, country 1 achieves the highest welfare level when it sets its unconstrained carbon tax, γd, and attracts both firms. However, by comparing (15b) and (15c), it is no more obvious to say which welfare level is higher. Although the consumer and producer surplus are larger under NR than under PR_1, marginal damages are also larger. Furthermore, there is a new source of income, i.e. the BCAs revenues, which country 1 would collect only under PR_1. It is clear from Fig. 4, that the two welfare level intersects at a higher t_2 than under the previous regime.

Despite being equipped with the BCA-policy, country 1 still has the incentive to undercut the other country's tax rate in order to attract both firms if $t_2 \leq \gamma d$. However, with BCAs country 1 stops undercutting earlier than under the No-BCA-regime, where it now prefers to set a higher tax rate than country 2 such that its home firm partially relocates. Therefore, there exists a tax level t_2, at which country 1 is indifferent between two best responses: to undercut t_2 and have both firms or to set a higher tax level than t_2 and keep only one plant. We denote this critical tax level by $\underline{t_2}(\gamma, d, A)$, which is derived in Appendix B.2. At this level, both $\tilde{W}_1^{TR_2}$ and $\hat{W}_1^{PR_1}$ intersect as shown in Fig. 4.[19]

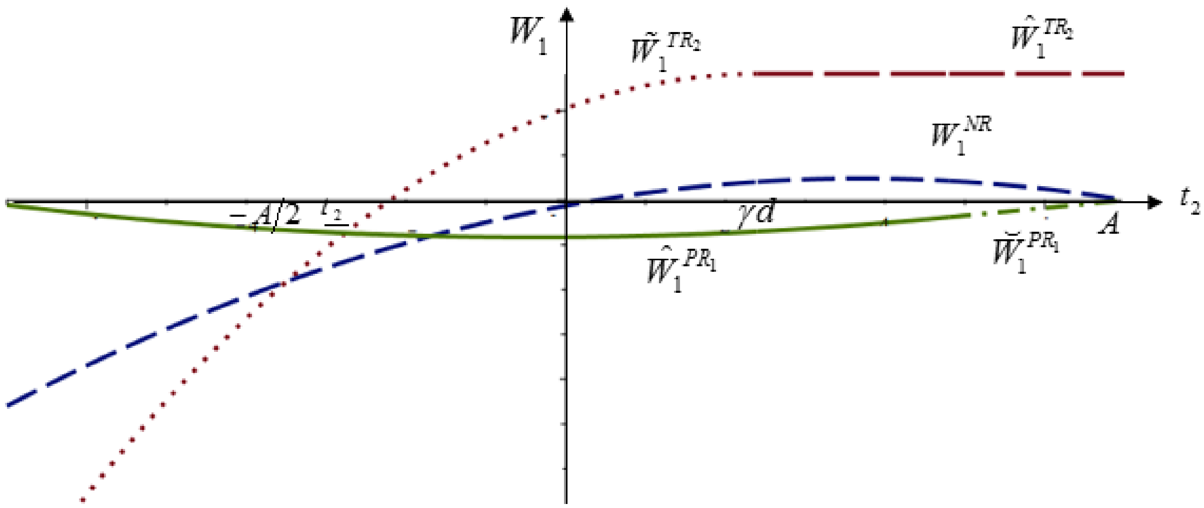

Fig. 4: Ranking of the Welfare Levels of Country 1 with BCAs

It is important now to understand the incentives of country 1 at this switch point. On the one hand, undercutting implies that all plants are located in country 1. This leads to a higher consumer and producer surplus. However, since $\underline{t_2} < 0$, country 1 incurs costs of subsidising the two firms. On the other hand, by imposing a tax rate higher than t_2, more precisely, $\hat{t}_1^{PR_1}$, country 1 earns carbon tariff revenues from the foreign plant, in addition to the tax revenues obtained from its domestic plant. Furthermore, it suffers less damages under PR_1 because both plants supplying country 1 face a higher carbon tax. These opposing effects cancel each other at $\underline{t_2}$, such that gains achieved under PR_1 dominate for all $t_2 < \underline{t_2}$.

At this point, we also need to explain how the parameters of the model affect the decision of country 1 to stop undercutting. We find that $\underline{t_2}$ increases in γ and d, i.e. in the marginal damages of country 1. The larger γd, the larger $\hat{t}_1^{PR_1}$, hence, the larger environmental benefits and tariff revenues under PR_1. This reduces the incentive of country 1 to undercut t_2, as a result, it switches to PR_1 at a higher level of $\underline{t_2}$. On the other hand, $\underline{t_2}$ decreases in A. As the market size increases, higher consumer surplus and larger profit opportunities under TR_2 outweigh gains that would be achieved from BCAs. Hence, undercutting stops at a low level of $\underline{t_2}$.

Lemma 2. The Best Response of Country 1 under the BCA-Regime

[19]The Welfare functions in Fig. 4 correspond to case $(2(a))$ of Lemma 2, which is provided in Appendix B.2. However there will not be a qualitative change in the figure if we consider other cases.

i. If $\gamma d < t_2 < A$, the welfare level of country 1 can be ranked as $\hat{W}_1^{TR_2} > W_1^{PR_1}$ and $\hat{W}_1^{TR_2} > W_1^{NR}$.

ii. If $\underline{t_2} < t_2 \leq \gamma d$, the welfare level of country 1 can be ranked as $\tilde{W}_1^{TR_2} > W_1^{PR_1}$ and $\tilde{W}_1^{TR_2} > W_1^{NR}$.

iii. If $-\frac{1}{2}A < t_2 \leq \underline{t_2}$, the welfare level of country 1 can be ranked as $\hat{W}_1^{PR_1} \geq \tilde{W}_1^{TR_2} > W_1^{NR}$

iv. If $t_2 \leq -\frac{1}{2}A$, the welfare level of country 1 can be ranked as $\hat{W}_1^{PR_1} > W_1^{NR} \geq \tilde{W}_1^{TR_2}$.

Hence, the best response of country 1 is given by:

$$t_1 \begin{cases} = \gamma d & \text{if } \gamma d < t_2 < A \\ = t_2\text{-}\varepsilon & \text{if } \underline{t_2} < t_2 \leq \gamma d \\ = \hat{t}_1^{PR_1}(t_2) > t_2 & \text{if } t_2 < \underline{t_2} \end{cases} \tag{21}$$

Proof. See Appendix B.2 for the sub-cases of Lemma 2, and the proof of each case. □

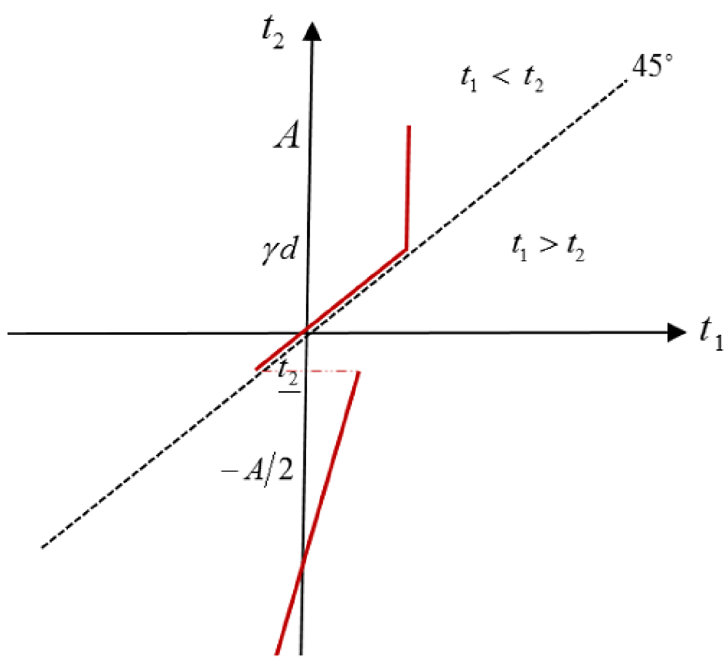

Fig. 5: Reaction Function of Country 1 with BCAs

Under the BCA-regime, the reaction function of country 1 also has three parts. The first two parts are similar to the previous regime: a vertical line, i.e. the dominant strategy, and the undercutting part, which lies slightly above the 45° line. Different from Fig. 2a, the third part is upward sloping, which is the standard reaction function under PR_1. Fig. 5 shows two important features: firstly, the reaction function is discontinuous at $t_2 = \underline{t_2}$, where country 1 is indifferent between two best responses as explained above. Secondly, matching taxes with country 2 is not a best response at any given level of t_2. As we will show later, these two features greatly affect the NE under this regime.

Turning to country 2, the welfare functions are defined below, where the first case only is new in this regime.

1. In the case $t_1 > t_2$:
   (a) Let $\hat{W}_2^{PR_1}$ be $W_2^{PR_1}\left(t_2 = \hat{t}_2^{PR_1}(t_1)\right)$ as a function of t_1, in the range $t_1 > \bar{t}_1$, i.e. country 2 sets the tax level in (20).
   (b) Let $\tilde{W}_2^{PR_1}$ be $W_2^{PR_1}(t_2 = t_1 - \varepsilon)$ as a function of t_1, in the range $t_1 \leq \bar{t}_1$, i.e. country 2 undercuts the tax of country 1.
2. In the case $t_2 = t_1$, we have W_2^{NR} as defined in the No-BCA-regime.
3. In the case $t_1 < t_2$, we have $W_2^{TR_2}$ which is also defined in the No-BCA-regime.

Inserting the previously defined tax levels into (19), we obtain $\hat{W}_2^{PR_1}$ and $\tilde{W}_2^{PR_1}$. See Appendix B.3 for the details.

The incentives of country 2 are similar to the previous regime. In Fig. 6, it is clearly shown that for all $t_1 > \bar{t}_1$, country 2 achieves the highest welfare level if it sets its optimal tax level, $\hat{t}_2^{PR_1}$, to attract the three plants. However, since $\hat{t}_2^{PR_1}$ is a function of t_1, we can notice that $\hat{W}_2^{PR_1}$ is a convex function compared to $\hat{W}_2^{TR_1}$, which is a constant function, i.e independent of t_1.

As t_1 decreases, i.e. if $t_1 \leq \bar{t}_1$, country 2 responds by a constrained carbon tax, i.e by undercutting t_1 under PR_1, hence $\tilde{W}_2^{PR_1}$ is a concave function. As we explained in the No-BCA-regime, undercutting will stop when profits minus subsidies become zero, i.e. at $t_1 = -\frac{1}{2}A$. That is, further undercutting means too much subsidy that outweighs profits of firms, hence it is not a best response.

From the previous analysis, the best response of country 2 is summarised in the following Lemma.

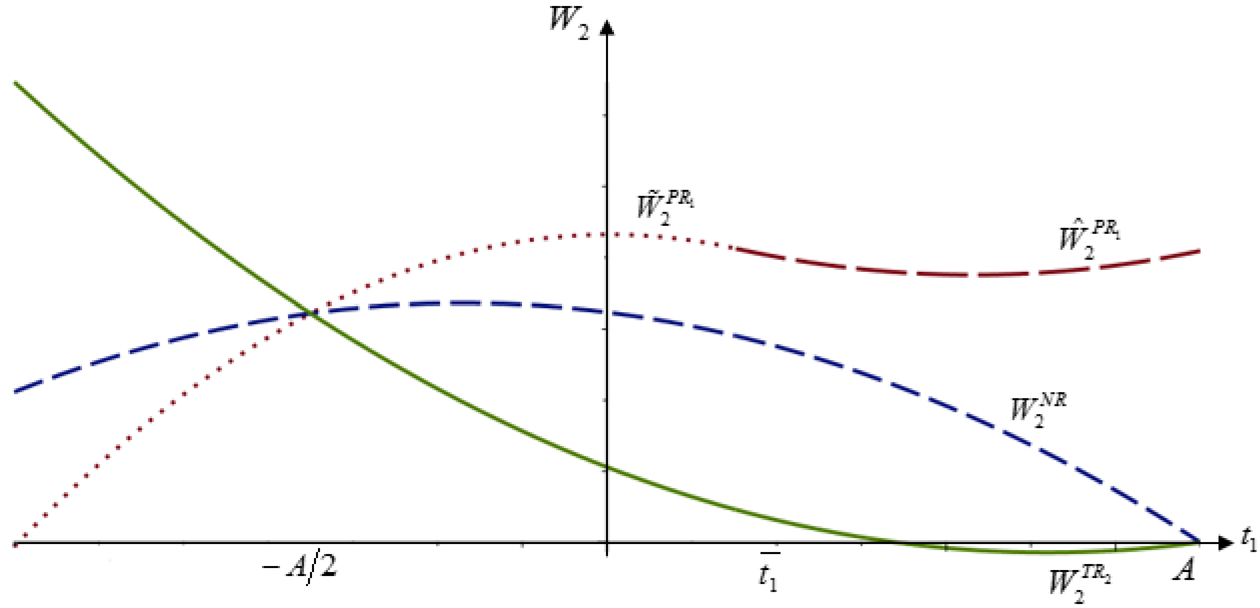

Fig. 6: Ranking of the Welfare Levels of Country 2 with BCAs

Lemma 3. The Best Response of Country 2 under the BCA-Regime

i. If $\bar{t}_1 < t_1 < A$, the welfare level of country 2 can be ranked as follows: $\hat{W}_2^{PR_1} > W_2^{NR} > W_2^{TR_2}$.

ii. If $-\frac{1}{2}A < t_1 \leq \bar{t}_1$, the welfare level of country 2 can be ranked as follows: $\tilde{W}_2^{PR_1} > W_2^{NR} > W_2^{TR_2}$.

iii. If $t_1 \leq -\frac{1}{2}A$, the welfare level of country 2 can be ranked as follows:$W_2^{TR_2} \geq W_2^{NR} \geq \tilde{W}_2^{PR_1}$.

Hence, the best response of country 2 is given by:

$$t_2 \begin{cases} = \hat{t}_2^{PR_1}(t_1) & \text{if } \bar{t}_1 < t_1 < A \\ = t_1\text{-}\varepsilon & \text{if } -\frac{1}{2}A < t_1 \leq \bar{t}_1 \\ \geq t_1 & \text{if } t_1 \leq -\frac{1}{2}A \end{cases} \tag{22}$$

Proof. See Appendix B.4. □

The reaction function of country 2 in Fig. 7 is nearly the same as in the No-BCA-regime, except for the range $\bar{t}_1 < t_1 < A$, where it becomes downward sloping, i.e. function of t_1, compared to a horizontal line in Fig. 2b.

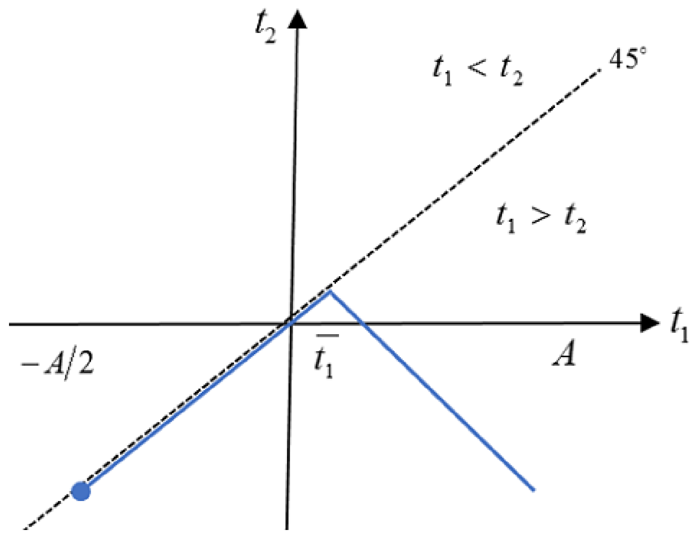

Fig. 7: Reaction Function of Country 2 with BCAs

4.1 Simultaneous Game

The fact that country 1 stops undercutting before country 2 under the BCA-regime implies that a competition towards the bottom is eliminated. Hence, unlike the previous regime, a 'race to the bottom' is not a NE after introducing the BCA-policy. In addition, since matching taxes is never a best response for country 1 with BCAs, NR does not emerge as an equilibrium. Therefore, if a NE exits, it is that country 1 sets a higher tax than country 2 and firm 1 partially relocates to country 2. This case is illustrated in Fig. 8a, where the two standard reaction functions under PR_1 intersect. Solving (18) and (20) simultaneously, we obtain the NE under PR_1 as follows:[20]

$$t_1^{*NE}(PR_1) = \frac{1}{11}A + \frac{1}{11}d\,(6 + 2\gamma) \tag{23}$$

$$t_2^{*NE}(PR_1) = \frac{2}{11}A + \frac{2}{11}d\,(6 - 9\gamma) \tag{24}$$

However, a NE does not always exist. As mentioned by Mintz and Tulkens (1986), this could arise due to the multivaluedness of the reaction functions. The jump in

[20]Note that if we consider the location equilibrium PR_1 in a fixed plant location model, we will always have a unique NE since $\frac{\partial^2 W_1^{PR_1}}{\partial t_1^2}\frac{\partial^2 W_2^{PR_1}}{\partial t_2^2} - \left(\frac{\partial^2 W_1^{PR_1}}{\partial t_1 \partial t_2}\frac{\partial^2 W_2^{PR_1}}{\partial t_2 \partial t_1}\right) = \frac{11}{27} > 0$, which ensures existence and uniqueness of a NE. However, as shown in Fig. 8b, a NE does not always exist under endogenous plant location model.

the reaction function of country 1 implies that at $\underline{t_2}$, there are two values of t_1, which gives country 1 the same welfare level, i.e. two best responses.[21]

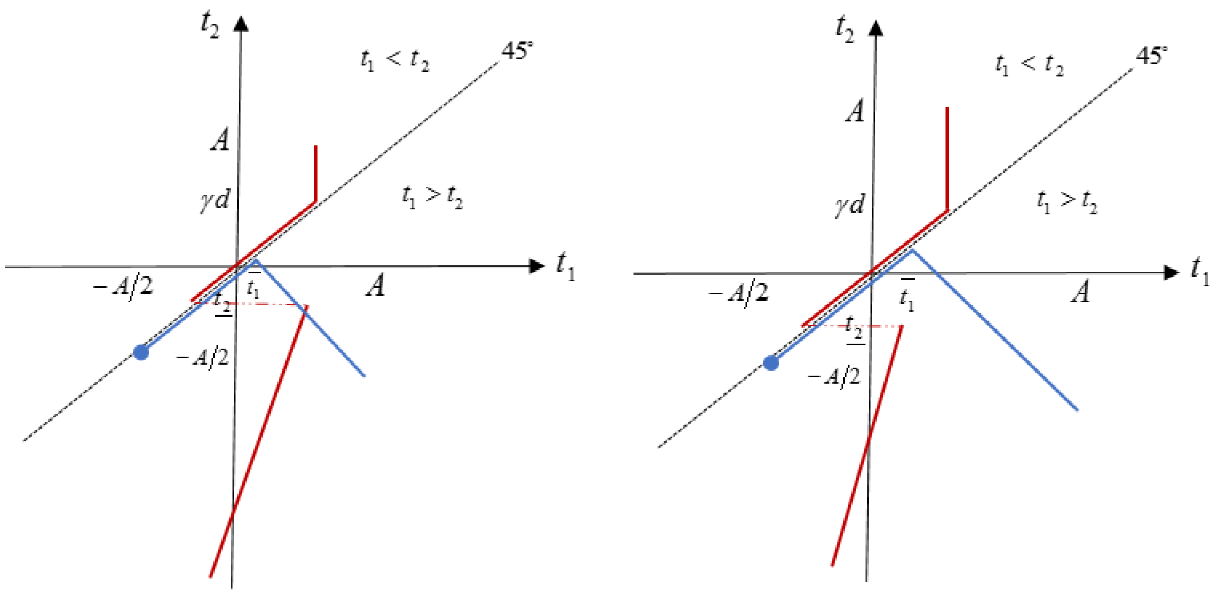

Fig.8(a) Existence of a NE

Fig8.(b) Non-existence of a NE

Fig. 8: Nash Equilibrium with BCAs

For a pure strategy NE to exist, the intersection of the reaction functions under PR_1 must occur in the range where country 1's best response is on its standard reaction function. This means that we must have $\underline{t_2} \geq t_2^{*NE}(PR_1)$. However, if $\underline{t_2} < t_2^{*NE}(PR_1)$, the best response of country 1 would be undercutting, hence, reaction functions do not intersect and no pure strategy NE exists as illustrated in Fig. 8b.

Proposition 5. Nash Equilibrium under the BCA-Regime

i. A pure strategy NE may or may not exist.

ii. A pure strategy NE exists if global marginal damages and asymmetry among countries are sufficiently large.

iii. If a NE exists, we have $t_1^{*NE}(PR_1) > t_2^{*NE}(PR_1)$, where $t_1^{*NE}(PR_1) > 0$ and $t_2^{*NE}(PR_1) < 0$, and the location equilibrium of firms is PR_1.

Proof. See Appendix B.5. □

To understand the driving forces behind the existence of such an equilibrium, we find that $\underline{t_2} \geq t_2^{*NE}(PR_1)$ is true if $A \leq \overline{A}_{NE}$. This threshold level of market size increases

[21]See also Anderson (1987).

with d and γ. That is, if marginal damages of country 1 are sufficiently high, i.e. if $\gamma > 0.855$, a NE may exist. In our model, this implies that marginal damages of country 2 are sufficiently low. In such cases, benefits from larger tariff revenues and reduction of damages allows country 1 to switch to PR_1 at a higher $\underline{t_2}$. In addition, $t_2^{*NE}(PR_1)$ is decreasing in γ. Therefore, the larger the global marginal damages and the asymmetry among countries, the larger the gap between $\underline{t_2}$ and $t_2^{*NE}(PR_1)$, and the more likely a NE exists.

One the one hand, Proposition 5 shows that if global marginal damages are too high, BCAs may be an effective policy to mitigate the negative impacts resulting from the fierce competition in carbon taxes to attract firms. It is interesting that in such cases, the environmental costs associated with the 'race to the bottom' equilibrium become very large compared to the social optimum outcome. On the other hand, since a NE does not always exists, BCAs on imports only may not be sufficient to lead to a better outcome. Therefore, a stronger policy might be needed to support the existence of a NE under more general conditions.

4.2 Sequential Game

We now solve the first stage in the second policy game under the BCA-regime. *Firstly*, if country 1 leads, it considers the reaction function of the follower given in (22). Country 1 cannot attract firm 2 for all $t_1 > -\frac{1}{2}A$, and it has no incentive to attract both firms for all $t_1 < -\frac{1}{2}A$. As a result, there are basically three choices for country 1: *(i)* set $t_1^L = -\frac{1}{2}A$ and keep its home firm, i.e. NR *(ii)* impose BCAs, and set $t_1^L > \bar{t}_1$ to induce relocation of one plant of its home firm, i.e. PR_1 and *(iii)* set $t_1^L > (1-\gamma)\, d$ without imposing BCAs to induce total relocation of its home firm, i.e. TR_1.[22]

As will be shown later in Appendix C.2, the last choice is always dominated by the second choice, hence country 1 will not allow the two plants to relocate. Therefore, country 1 chooses between NR and PR_1. As we have explained in subsection 3.2, the SE under NR is $t_1^{*L} = t_2^{*F} = -\frac{1}{2}A$. Thus, what is remaining here is to derive the SE under PR_1.

As a leader, country 1 maximises its welfare under PR_1 taking into account the best response of country 2, i.e. maximises $W_1^{PR_1}\left(t_1, \hat{t}_2^{PR_1}(t_1)\right)$ with respect to t_1. This gives the equilibrium carbon tax of the leader and the follower, respectively, under PR_1 as follows:

$$t_1^{*L}(PR_1) = \frac{2}{7}A + \frac{1}{7}d\,(3-2\gamma) \tag{25}$$

$$t_2^{*F}(PR_1) = \frac{1}{28}A + \frac{1}{7}d\left(\frac{33}{4} - 9\gamma\right) \tag{26}$$

[22] We find that country 1 always prefers country 2 to react by choosing its optimal tax under PR_1 not by undercutting. See the proof of Proposition 6.

where $t_1^{*L}(PR_1) > \bar{t}_1$. Hence, if country 1 chooses to impose BCAs, the SE is given by (25) and (26). Facing the choices mentioned above, the two Stackelberg equilibria under the BCA-regime are stated in the following proposition.

Proposition 6. Stackelberg Equilibrium under the BCA-Regime if Country 1 Leads

i. Equilibrium 1 is a subsidy, $t_1^{*L} = t_2^{*F} = -\frac{1}{2}A$*, and the equilibrium location of firms is* NR*, if* $A > \hat{A}_1^{PR_1}$*.*

ii. Equilibrium 2 is $t_1^{*L}(PR_1) > t_2^{*F}(PR_1)$*, and the equilibrium location of firms is* PR_1*, if* $A < \hat{A}_1^{PR_1}$*.*

iii. $\hat{A}_1^{PR_1}$ *increases in* γ *and* d*.*

Proof. See Appendix B.6. □

There exists a threshold of market size, $\hat{A}_1^{PR_1}$, under which country 1 would prefer partial relocation of its home firm. Again, this SE allows the leader to escape the 'race to the bottom'. The larger the asymmetry among countries, the more likely country 1 would choose PR_1. Different from the simultaneous game, if country 1 leads, its equilibrium carbon tax decreases in γ. Hence, from (17), both the consumer and producer surplus become larger under PR_1 as γ increases. In addition, the difference between carbon taxes of countries increases with γ, which pushes the BCAs revenues higher. Therefore, these gains outweigh higher consumer surplus under NR which makes it more likely that a PR_1 be chosen. However, Proposition 6 shows that under some conditions, where global marginal damages are not large, BCAs may be ineffective in supporting more strict climate policies. This also confirms the results obtained in the simultaneous game.

Secondly, if country 2 leads, it takes into account the best response of country 1 given in (21). It is clear here that for any $\underline{t_2} < t_2$, country 2 is unable to have both firms within its territory. Therefore, there are only two choices for country 2: *(i)* set $t_2 < \underline{t_2}$ and attract one plant of the foreign firm, in addition to its home firm, i.e. PR_1, or *(ii)* set $t_2 > \underline{t_2}$ to induce total relocation of its home firm, i.e. TR_2. Two main differences need to be pointed out after introducing the BCA-policy. First, as a leader, country 2 cannot attract any plant of the foreign firm without BCAs. Second, the choice to set $t_2 = -\frac{1}{2}A$ is no longer available with BCAs, hence NR, or equivalently the 'race to the bottom', never emerge as a SE under leadership of country 2 in this regime.

In the following, we consider the first option for country 2. As a leader, it maximises its welfare level under PR_1, taking into account the best response of country 1, i.e. $W_2^{PR_1}\left(t_2, \hat{t}_1^{PR_1}(t_2)\right)$. The equilibrium carbon tax of the leader and the follower are given respectively by:

$$t_2^{*L}(PR_1) = \frac{1}{13}d\left(18 - 22\gamma\right) \tag{27}$$

$$t_1^{*F}(PR_1) = \frac{d\,(2\gamma+9)}{13} \tag{28}$$

However, for country 2 to set its welfare-maximising carbon tax, $t_2^{*L}(PR_1)$ should be less than $\underline{t_2}$, otherwise, country 1 would undercut t_2 to have both firms. Therefore, as we did in the simultaneous game, we find that for $t_2^{*L}(PR_1) < \underline{t_2}$ to be true, we must have $A < \overline{A}_{SE}$, which is a comparable to the condition of existence of a NE. [23] This condition is only feasible if countries are highly asymmetric, i.e. if $\gamma > \widehat{\gamma} = 0.881$. Thus, only if global marginal damages are high and countries are sufficiently asymmetric, country 2 could impose $t_2^{*L}(PR_1)$. The intuition is the same as before. That is, if countries are highly asymmetric, i.e. if γ is large, $t_2^{*L}(PR_1)$ decreases, while $\underline{t_2}$ increases. Thus, the gap between the two tax levels becomes smaller which may allow $t_2^{*L}(PR_1) < \underline{t_2}$ to be true.

However, if these conditions do not hold, country 2 must deviate from (27) if it would be better off under PR_1. If country 2 sets its tax level equal to $\underline{t_2}$, country 1 will be indifferent between undercutting or setting a higher tax rate, i.e. $\hat{t}_1^{PR_1}$. If country 2 is better off under PR_1, it will choose the highest possible tax level, which induces country 1 to react on its standard reaction function under PR_1. That is, country 2 sets a tax level marginally below $\underline{t_2}$, i.e. $t_2^{*L} \lesssim \underline{t_2}$.[24] However, if country 2 prefers the TR_2 equilibrium, it will set a tax level marginally above $\underline{t_2}$, i.e. $t_2^{*L} \gtrsim \underline{t_2}$, such that country 1 responds by undercutting this tax level.

Proposition 7. Stackelberg Equilibrium under the BCA-Regime if Country 2 Leads

i. Equilibrium 1 is (a) $t_2^{*L}(PR_1) < t_1^{*F}(PR_1)$ *if* $A < \overline{A}_{SE}$ *for all* $\gamma > \widehat{\gamma}$ *(b)* $t_2^{*L} \lesssim \underline{t_2} < \hat{t}_1^{PR_1} = t_1^{*F}$ *if* $\overline{A}_{SE} < A < \overline{A}^{TR_2}$ *for all* $\gamma > \widehat{\gamma}$ *or if* $A < \overline{A}^{TR_2}$ *for all* $\gamma < \widehat{\gamma}$*. The equilibrium location of firms is* PR_1*.*

ii. Equilibrium 2 is $t_2^{*L} \gtrsim t_1^{*F} \gtrsim \underline{t_2}$*, and the equilibrium location of firms is* TR_2*, if* $A > \overline{A}^{TR_2}$*.*

Proof. See Appendix B.7. □

We show in Proposition 7 that if country 2 is able to choose its welfare-maximising tax level under PR_1, i.e. if global marginal damages are sufficiently large, it will choose this tax level and attract the foreign plant. Country 2 also have the incentive to set its constrained carbon tax under PR_1, i.e. $t_2^{*L} \lesssim \underline{t_2}$ and have the three plants

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[23] $\overline{A}_{SE} = \frac{d\left(11\gamma - 9 + 3\sqrt{185\gamma^2 - 246\gamma + 90}\right)}{13} + \varepsilon$

[24] For a similar case, see Dixit (1979), though in a different context. He explains that if the leader chooses its strategy at the point where the follower is indifferent between two best responses, there is technically no equilibrium if the leader prefers one response by the follower to the other. In such cases, the leader can choose its strategy slightly larger or smaller than this indifference point such that the follower responds in a way which gives the leader the highest benefits.

within its national boundaries. However, if global marginal damages are too low, country 2 may prefer to let its firm to relocate, which is the second equilibrium. In such cases, $\underline{t_2}$, which is a subsidy level, becomes large, while profits of the home plant, which faces carbon tariffs decrease. As a result, some portion of the profits of firm 2 goes to country 1 through tariffs such that total profits may not compensate for the subsidy cost in country 2. However, in such cases, country 2 obliges the follower's country to set its carbon tax slightly above $\underline{t_2}$ to protect the consumers. Hence, the incentives of country 2 to choose the second equilibrium is different from the No-BCA-regime. In this equilibrium, BCAs act as a threat that allow country 1 to attract both firms but will not be imposed.

4.3 Comparisons under the BCA-Regime

In this subsection, we compare the non-cooperative taxes under the BCA-regime with the socially optimal tax. However, since a NE does not always exist, we will not compare the climate policies across the two policy games.

Corollary 2. *Under the BCA-regime, if global marginal damages are low relative to the market size:*

i. Country 1 may impose a carbon tax larger than the socially optimal tax in both the simultaneous and the sequential game.

ii. Country 2 may impose a carbon tax larger than the socially optimal tax in the sequential game only if country 1 is the Stackelberg leader.

iii. Global emissions may be less only in the sequential game compared to the social optimum.

Proof. See Appendix B.8. □

Different from the No-BCA-regime, countries, in particular country 1, may adopt stricter climate policies than under the social optimum when BCAs are introduced. With BCAs, country 1 has an incentive to extract revenues from country 2 through carbon tariffs, thus, country 1 may impose a carbon tax larger than under the social optimum. The above results show that the strategic role of BCAs to push country 2 to set a larger carbon tax is more effective if country 1 acts as the Stackelberg leader and chooses the PR_1 equilibrium. The intuition here is that for country 2, taxes are strategic substitutes. Taking this into account, country 1 as a leader sets its tax rate below the NE level to induce country 2 to choose a carbon tax above the NE. However, despite these positive results on the effectiveness of BCAs to reduce global emissions, Corollary 2 shows that if global damages are large, the lowest global emissions level would be achieved only under the social optimum solution.

5 Comparisons of Climate Policies across Regimes: The Role of BCAs

In this section, we compare the previously derived equilibrium outcomes under the two alternative regimes to analyse the effect of adding the BCA-policy. In the following two subsections, we compare both regimes given a sequence of move, i.e. we do the comparison for the simultaneous and the sequential game, respectively. In this section, most of our results are expressed in a space of two parameters: γ, which reflects the degree of asymmetry among countries, and a newly defined parameter, β, which is the ratio of the market size to the global marginal damage, i.e. $\beta = \frac{A}{d}$.

5.1 Simultaneous Game

To evaluate the effects of BCAs in this game, we will consider the range of parameters under which a pure strategy NE exists under the BCA-regime. Taken together, Proposition 2 and 5 allow us to draw the following conclusions.

Corollary 3. The Effect of BCAs on Climate Policies and Equilibrium Location in the Simultaneous Game.

i. BCAs lead to more stringent climate policies in both countries, which lead to less global emissions.

ii. BCAs change the equilibrium location of firms from NR to PR_1.

Proof. For country 1, the equilibrium policy under the BCA regime, $t_1^{*NE}(PR_1)$ is always a tax, while under the No-BCA regime $t_1^{*NE} = -\frac{1}{2}A$ is a subsidy. For country 2, the equilibrium policy under the BCA-regime, $t_2^{*NE}(PR_1)$ is larger than its policy under the No-BCA-regime, $-\frac{1}{2}A$, as long as the NNC is satisfied. □

As shown above, BCAs eliminate the 'race to the bottom'. That is, it allows both countries to choose higher carbon taxes than under the No-BCA-regim. By imposing BCAs, country 1 would be able to prevent total relocation of its firm and, additionally, collects tariff revenues of the foreign firm. Country 2 would also impose a higher carbon tax without the fear of being undercut by country 1. As a result, although BCAs induce a partial relocation of firm 1, this leads to less global emissions.

The following corollary looks at the effect of BCAs on the welfare levels, and explains under which conditions countries may gain or loose from this additional policy.

Corollary 4. The Effect of BCAs on Individual and Global Welfare Levels in the Simultaneous Game

With BCAs : i. Country 1 welfare increases.

ii. Country 2 welfare increases if its marginal damages are not too low.

iii. Global welfare increases.

Proof. See Appendix C.1. □

Under the No-BCA-regime, both countries set the maximum subsidy level such that profits minus subsidies is zero. However, with BCAs, country 1 gains tax and tariff revenues in addition to profits of one plant. Furthermore, it enjoys benefits of less damages. These gains outweigh the loss in consumer surplus due to the higher taxes under PR_1. Hence, the welfare level of country 1 increases with BCAs. From the prescriptive of country 2, profits minus subsidies also become positive with BCAs. Additionally, it enjoys benefits from less global emissions. These positive effects compensate for the loss in consumer surplus for all $\gamma < 0.96$. However, as γ increases, or alternatively as its marginal damages decrease, two factors contribute to its welfare loss under the BCA-regime. First, reduction of damages becomes less important for country 2. Second, the difference between the two national tax rates under PR_1 becomes larger. Hence, profits of its firm, which serves country 1 fall due to facing a larger carbon tariff, consequently, its tax revenues decrease. As a result, if countries are very highly asymmetric, the welfare level of country 2 may decrease with BCAs. More importantly, we find that introducing the BCA-policy raises the global welfare level. However, it is important to note that conclusions drawn in this subsection are limited to the range of parameters under which a NE exits with BCAs, i.e. if both global marginal damages and asymmetry among countries are large.

5.2 Sequential Game

In order to analyse the effect of BCAs in the sequential game, we divide the feasible range of values of parameters into regions, which are illustrated in Fig. 9. As we did before, we will consider the two sequential orders in turn. If country 1 leads, we have three regions, F, G and H, shown in Fig. 9a.

Corollary 5. The Effect of BCAs if Country 1 Leads in the Sequential Game.

i. BCAs change the equilibrium location of firm 1 from TR_1 to PR_1 in region F, and from NR to PR_1 in region G, which lead to less global emissions. This is associated with higher global welfare if the asymmetry among countries is not too high or if global marginal damages are large and countries are highly asymmetric.

ii. BCAs do not affect the equilibrium location of firms in region H, which remains NR, that leads to the same global emissions and global welfare level.

Proof. See Appendix C.2. □

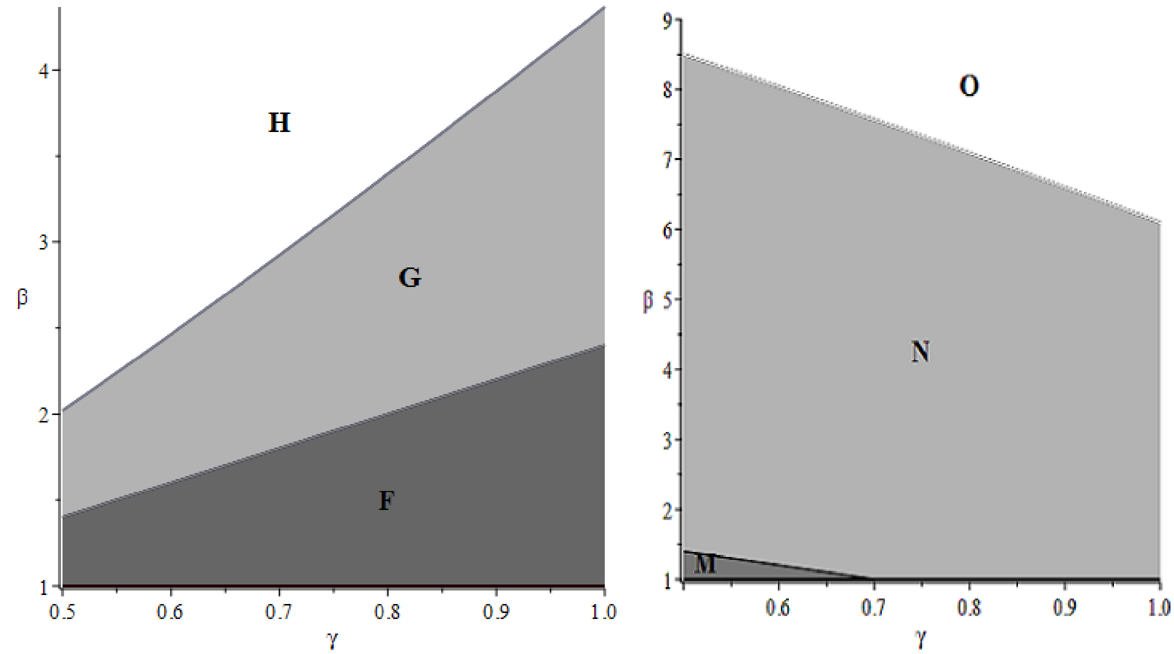

Fig.(9a) Country 1 Leads

Fig.(9b) Country 2 Leads

Fig. 9: Equilibrium Outcomes under Alternative Regimes in the Sequential Game*
*$\beta = \frac{A}{d}$

In region F, although firm 1 relocates under both regimes, it relocates only partially under the BCA-regime which leads to less global emissions. Without BCAs, all firms are located in country 2, which sets its optimal carbon tax $(1-\gamma)\,d$. However, the two plants supplying country 1 face a larger carbon tax under the BCA-regime. Furthermore, both plants supplying country 2 would also face a higher carbon tax if $\gamma < 0.75$. Hence, global emissions are reduced. From an individual perspective, BCAs have opposite impacts on countries. One the one hand, this policy allows country 1 to enjoy benefits accrued from: profits of one home plant, tax and carbon tariff revenues and less global emissions. These gains offset the loss in consumer surplus due to a higher carbon tax. On the other hand, country 2 suffers from loosing one foreign plant, in addition to a reduction in profits of its home plant, which faces a carbon tariff at the border. Consequently, its tax revenues also decrease. These negative effects exceed the gains achieved from lower damages, in particular if its marginal damages become low. As a result, the effect of BCAs on global welfare is ambiguous. However, we find that if the asymmetry among countries is not too high or if global marginal damages are large and countries are highly asymmetric, the welfare gains by country 1 may compensate for the loss incurred by country 2, hence global welfare may increase.

In region G, BCAs change the location equilibrium of firms from NR to PR_1. Hence, country 1 is more likely to avoid the 'race to the bottom' equilibrium under the BCA-regime. In such cases, both countries impose a higher carbon tax which results in less global emissions. It is obvious here that country 1 is better off with BCAs. Unlike the previous region, BCAs allow country 2 to attract one foreign plant to its territory, in addition to the benefits received from less global emissions. However, consumers are still disadvantaged since they face higher prices. The net effect is an increase in the welfare level of country 2 unless its marginal damages become too low, i.e. $\gamma > 0.98$.

Finally, in region H, the location equilibrium of firms does not change, i.e. NR under the two regimes. Hence, the BCA-policy is ineffective in this region and countries cannot weed the 'race to the bottom' out. Consequently, global emissions and global welfare level remain the same. However, and as clearly shown in Fig.9a, this result arises when global marginal damages are not large.

Corollary 6. The Effect of BCAs if Country 2 Leads in the Sequential Game

i. BCAs change the equilibrium location of firms from TR_2 to PR_1 in region M, which leads to higher global emissions and a lower global welfare level.

ii. BCAs change the equilibrium location of firms from NR to PR_1 in region N,which leads to higher global welfare, and from NR to TR_2 in region O, which leads to higher global welfare level if global marginal damages are not too low. In addition, BCAs lead to lower global emissions.

Proof. See Appendix C.3. □

In Fig. 9b, we compare the equilibrium outcomes if country 2 leads. In region M, firms change their equilibrium location from TR_2 to PR_1 with BCAs. In contrast to all previous results, this is the only case under which BCAs lead to larger global emissions. Without BCAs, both firms are located in the environmentally more concerned country, i.e. country 1, in which they face a tax equal to its marginal damages, i.e. γd. However, with the BCA-policy, both countries set a lower carbon tax which leads to higher global emission level. Furthermore, country 1 is always worse off with BCAs in this region since it looses profits and tax revenues of three plants. In addition, it suffers from larger environmental damages. From country 2's point of view, it becomes better off with BCAs. Besides the profits of firms, country 2 also gains from a higher consumer surplus. Although BCAs have opposite effects on individual welfare, global welfare level unambiguously decreases. However, note that even though this region of parameter values is very small, these negative results under the BCA-regime will not emerge. Due to the lower welfare level in country 1, BCAs are not credible in this region, hence country 1 will not choose to impose BCAs in such cases.

In region N, the equilibrium location of firms changes from NR to PR_1 with BCAs. Both countries subsidise their firms at $t_i = -\frac{1}{2}A$, under NR, while they impose a higher carbon tax with BCAs, which reduces global emissions. In addition, country 1 gains higher producer surplus and tariff revenues. Country 2 is also better off with BCAs, except the case if countries are highly asymmetric and global marginal damages are very low compared to the market size. As a result, global welfare increases with BCAs unless there is a substantial loss in the welfare level of country 2. However, global welfare level increases with BCAs in this region.

Finally, in region O, the location equilibrium becomes TR_2 compared to NR without BCAs. In this case, all firms are located in country 1. Although country 1 is constrained by setting a tax level marginally above $\underline{t_2}$ in this equilibrium, global emissions decrease and country 1 is better off. In addition, global welfare level increases if global marginal damages are not too low. In this region, BCAs act as credible threat to attract firms, but will not be imposed.

6 Conclusions

One main obstacle of more ambitious policies to address global warming are leakage effects. One particular form of leakage is the relocation of production, which may even imply that firms close down and move abroad if environmental regulation increases their production cost too much. This is in particular relevant for emission-intensive industries which trade internationally. In order to capture this phenomenon and the discussion surrounding it, we set up an intra-industry trade model and studied an emission tax competition game between two asymmetric countries when both, the location choice of firms and the policy choice of governments, are endogenous. Asymmetry implied in our model that countries evaluate the damages from global pollution differently. We solved a three-stage game in which governments first choose their emission tax, then firms choose the location of their two plants (one producing for the home and one for the foreign market), and finally firms choose their production levels. We considered two policy regimes. Under the No-BCA regime, each government imposes a carbon tax on the production within its national boundaries. This regime served as a benchmark to study the effect of border carbon adjustments, abbreviated BCAs, which we called the BCA-regime. Under this regime, the government which sets a higher carbon tax can additionally impose a tariff on imports, which have been produced facing a lower tax abroad. BCAs fully adjust the difference between the carbon taxes in the two countries. This implies that all plants supplying the country that imposes BCAs face the same effective carbon tax. As under the BCA-regime a Nash equilibrium (i.e., a simultaneous choices of taxes) in the tax game may not exist (due to discontinuity of reaction functions), we also considered Stackelberg equilibria (i.e., sequential choices of taxes).

Without BCAs, the effective taxes that firms face, and hence their profits, are based on the location of production. Thus, each firm will locate with its two plants in the country which sets a lower carbon tax. Thus, a government setting a higher carbon tax than its rival government will see its firm relocating abroad. If countries choose their climate policies simultaneously, this leads to a fierce tax competition in which each government has an incentive to undercut the other government's carbon tax in order to keep their firm or even attract the foreign firm. That is, we showed that the unique Nash equilibrium is a 'race to the bottom' leading to symmetric low taxes (or symmetric high subsidies) with high global emissions, irrespective of the absolute value of environmental damages and the degree of asymmetry among countries in terms of the evaluation of damages. In this equilibrium, each firm remains with its two plants in the country of origin, which we called no relocation (NR). Even if the environmentally more friendly government recognises damages as being important for the welfare of its country, it is rational for governments to lower taxes if the foreign government does so as well. By lowering taxes gradually, environmentally damages increase marginally, but the associated profit shifting effect is discrete, (i.e., avoiding the loss of profits because the own firm does not locate abroad and/or increasing profits because the foreign firm is attracted to the home country). This is the leakage dilemma of environmentally concerned governments. Interestingly, in a Stackelberg equilibrium with a sequential tax choice, governments may be able to avoid the 'race to the bottom' equilibrium. This is the case if the Stackelberg leader values environmental damages sufficiently high, recognises the disastrous outcome of the race to the bottom equilibrium and hence sets a high carbon tax such that its firm relocates to the follower's country. This total relocation (TR) equilibrium leads to less global emissions and, more importantly, is Pareto-improving for both countries compared to the Nash equilibrium.

Also under the BCA-regime, the pressure on the 'race to the bottom' was reduced even if governments choose their taxes simultaneously. A tariff allows the environmentally more concerned government to set a higher tax without the danger of loosing both plants. BCAs create an equal playing field for all production sold to the home market. Thus, higher taxes do not lead to total relocation (TR) but only to partial relocation (PR). Moreover, the government imposing tariffs at the border on imports gains a strategic advantage because it is able to shift tax revenues from abroad to home. In the simultaneous game, a Nash equilibrium may not exist (due to non-continuous reaction functions), though we showed that it exists when it is really needed. That is, it exists under those conditions where BCAs would raise global welfare substantially compared to the No-BCA regime. Also in the sequential game, it is more likely that the 'race to the bottom' is avoided with BCAs. Hence, taken together, our results showed that BCAs can support more ambitious climate policies and are globally welfare improving also under the more general assumption of endogenous plant location. However, we also demonstrated that BCAs will always fall short of achieving the socially optimal global welfare level.

In this paper, we considered a "weak form" of BCAs (also sometimes referred to as partial BCAs), which was a carbon tariff. However, also a "strong form " (also sometimes called full BCAs) has been suggested where tariffs are complemented by export rebates. That is, emission taxes on exports are reduced to the lower foreign emission tax level. This creates not only a equal playing field for goods sold to the market in which the higher emission tax is levied on production, but also for goods sold to the market in which the lower tax is imposed on production. De facto, this means to move from a production- to consumption-based emission tax. It may also imply that BCAs do not only avoid total relocation of firms located in environmentally friendly countries abroad, but may even avoid partial relocation. Hence, it would be interesting to explore whether "strong BCAs" could further improve global welfare. This is not obvious in a strategic context as export rebates per se increase pollution. Another extension could be to analyse whether and under which conditions BCAs are an effective policy tool to enforce a socially optimal solution. We showed that the social optimum can be obtained under three location equilibria, which greatly differ in the individual welfare levels of countries. In addition, BCAs result in partial relocation of firms from the more to the less environmentally concerned country. Hence, it is not straightforward to predict a priori whether BCAs would be a credible threat to enforce a socially optimal solution.

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A Appendix A

A.1 Possible Location equilibria

Consider the No-BCA-regime. As pointed out in section 2.1, firm k will base its decision where to locate its plant supplying market i on $\Delta\pi_{ki} = \pi_{ki}(i,\ell) - \pi_{ki}(j,\ell)$, $\ell = i, j$. For a given location of firm ℓ´s plant supplying market i, firm k will locate in country i if $\Delta\pi_{ki} > 0$ and will locate in country j if $\Delta\pi_{ki} < 0$. i) Let $\ell = i$. Then, $\Delta\pi_{ki} = \pi_{ki}(i,i) - \pi_{ki}(j,i) = -\frac{4}{9}(t_i - t_j)(A - t_j)$ as $\pi_{ki}(i,i) = \frac{(A-t_i)^2}{9}$ and $\pi_{ki}(j,i) = \frac{(A-2t_j+t_i)^2}{9}$, making use of (3) and (4) in the text, noting that the first payoff assumes tax vector (t_i, t_i) and the second (t_j, t_i). ii) Let $\ell = j$. Then, $\Delta\pi_{ki} = \pi_{ki}(i,j) - \pi_{ki}(j,j) = -\frac{4}{9}(t_i - t_j)(A - t_i)$ as $\pi_{ki}(i,j) = \frac{(A-2t_i+t_j)^2}{9}$ and $\pi_{ki}(j,j) = \frac{(A-t_j)^2}{9}$, again using(3) and (4) in the text and noting that the first payoff assumes tax vector (t_i, t_j) and the second (t_j, t_j). Since $A - t_j > 0$ and $A - t_i > 0$ by assumption, firm k's plant supplying market i has a dominant strategy, irrespective where firm ℓ´s plant supplying market i locates. It moves its plant to country j if $t_i > t_j$ and moves it to country i if the reverse is true, i.e., $t_i < t_j$. By assumption, if $t_i = t_j$, firm k´s plant supplying market i remains in its home country. Of course, taxes are faced on production and hence the same consideration applies for firm k´s plant supplying market j. Accordingly, all plants of firm k are based in the same country and hence there are three location equilibria: NR, TR_1 and TR_2. Under the BCA-regime, assuming that country i can impose BCAs, it is clear that the same computations apply for production sold to market j. For the protected market i, it is easily checked that $\Delta\pi_{ki} = 0$, irrespective where the competitor plant of firm ℓ supplying market i locates. Consequently, firm k with its plant supplying market i remains it its country of origin. If we let $i = 1$, then NR, PR_1 and TR_2 are possible location equilibria and if $i = 2$, then NR, PR_2 and TR_1.

A.2 The welfare functions of country 1 under possible location equilibria:

Recall that we assume $A > t_i$ and $A > d$ throughout the paper to ensure positive production levels in the possible location equilibria.

In the case $t_1 < t_2$: all plants are located in country 1 and face an effective tax $t_{k1} = t_{k2} = t_1$. Inserting these tax levels into (3) gives the equilibrium output levels under TR_2 as $x_{k1} = x_{k2} = (A - t_1)/3$.

Substituting these equilibrium output levels into (8a) gives the welfare level of country 1 under TR_2 as follows:

$$W_1^{TR_2} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_1\right)^2 + 4\left(\frac{A - t_1}{3}\right)^2 + t_1\left(\frac{4}{3}(A - t_1)\right) - \gamma d\left(\frac{4}{3}(A - t_1)\right) \tag{29}$$

where $W_1^{TR_2}$ is concave in t_1, $\frac{\partial^2 W_1^{TR_2}}{\partial t_1^2} = -\frac{4}{3} < 0$. Simplification of (29) leads to (9) in the text.

$\hat{W}_1^{TR_2}$ and $\tilde{W_1}^{TR_2}$ are obtained by inserting $t_1 = \gamma d$ and $t_1 = t_2 - \varepsilon$, respectively into (29) as follows:

$$\hat{W}_1^{TR_2} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}\gamma d\right)^2 + 4\left(\frac{1}{3}A - \frac{1}{3}\gamma d\right)^2 \tag{30}$$

$$\tilde{W_1}^{TR_2} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}(t_2 - \varepsilon)\right)^2 + 4\left(\frac{A - (t_2 - \varepsilon)}{3}\right)^2 + (t_2 - \varepsilon)\left(\frac{4}{3}(A - (t_2 - \varepsilon))\right) - \gamma d\left(\frac{4}{3}(A - (t_2 - \varepsilon))\right) \tag{31}$$

where $\tilde{W}_1^{TR_2}$ is also concave in t_2, $\frac{\partial^2 \tilde{W}_1^{TR_2}}{\partial t_2^2} = -\frac{4}{3}$.

In the case $t_1 = t_2$: each firm locates with its two plants in the country of origin and face an effective tax $t_{k1} = t_{k2} = t_1 = t_2$, which give the equilibrium output levels as $x_{k1} = x_{k2} = (A - t_1)/3 = (A - t_2)/3$.

Inserting these equilibrium output levels in (8b) gives:

$$\begin{aligned} W_1^{NR} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_1\right)^2 + 2\left(\frac{A - t_1}{3}\right)^2 + t_1\left(\frac{2}{3}(A - t_1)\right) \\ - \gamma d\left(\frac{4}{3}(A - t_1)\right) \end{aligned} \tag{32}$$

where $\frac{\partial^2 W_1^{NR}}{\partial t_1^2} = -\frac{4}{9} < 0$. Simplification of (32) gives (11) in the text.

Setting $t_1 = t_2$, equation (32) reads:

$$\begin{aligned} W_1^{NR} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_2\right)^2 + 2\left(\frac{A - t_2}{3}\right)^2 + t_2\left(\frac{2}{3}(A - t_2)\right) \\ - \gamma d\left(\frac{4}{3}(A - t_2)\right) \end{aligned} \tag{33}$$

In the case $t_1 > t_2$, all plants are located in country 2, and face an effective tax $t_{k1} = t_{k2} = t_2$. Inserting these tax levels into (3) gives the equilibrium output levels under TR_1 as $x_{k1} = x_{k2} = (A - t_2)/3$.

In this case, $W_1^{TR_1}$ is given by:

$$W_1^{TR_1} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_2\right)^2 - \gamma d\left(\frac{4}{3}(A - t_2)\right) \tag{34}$$

A.3 The welfare functions of country 2 under possible location equilibria:

Using the equilibrium output levels provided in Appendix A.2:

Inserting the equilibrium output levels in the case $t_1 > t_2$ into (8a), gives the welfare level of country 2 under TR_1 as follows:

$$\begin{aligned} W_2^{TR_1} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_2\right)^2 + 4\left(\frac{A - t_2}{3}\right)^2 + \\ t_2\left(\frac{4}{3}(A - t_2)\right) - (1 - \gamma)\, d\left(\frac{4}{3}(A - t_2)\right) \end{aligned} \tag{35}$$

where $W_2^{TR_1}$ is concave, $\frac{\partial^2 W_2^{TR_1}}{\partial t_2^2} = -\frac{4}{3} < 0$, and its simplification also leads to (9).

Inserting $t_2 = (1-\gamma)\,d$ and $t_2 = t_1 - \varepsilon$ into (35) gives $\hat{W}_2^{TR_1}$and $\tilde{W}_2^{TR_1}$, respectively as follows:

$$\hat{W}_2^{TR_1} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}(1-\gamma)\,d\right)^2 + 4\left(\frac{1}{3}A - \frac{1}{3}(1-\gamma)\,d\right)^2 \tag{36}$$

$$\begin{aligned}\tilde{W_2}^{TR_1} = {} & \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}(t_1-\varepsilon)\right)^2 + 4\left(\frac{A-(t_1-\varepsilon)}{3}\right)^2 + \\ & (t_1-\varepsilon)\left(\frac{4}{3}(A-(t_1-\varepsilon))\right) - (1-\gamma)\,d\left(\frac{4}{3}(A-(t_1-\varepsilon))\right)\end{aligned} \tag{37}$$

where $\tilde{W_2}^{TR_1}$ is concave in t_1.

Inserting the equilibrium output levels in the case $t_1 = t_2$ into (8b), gives the welfare level of country 2 under NR as follows:

$$\begin{aligned}W_2^{NR} = {} & \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_2\right)^2 + 2\left(\frac{A-t_2}{3}\right)^2 + t_2\left(\frac{2}{3}(A-t_2)\right) \\ & - (1-\gamma)\,d\left(\frac{4}{3}(A-t_2)\right)\end{aligned} \tag{38}$$

where $\frac{\partial^2 W_2^{NR}}{\partial t_2^2} = -\frac{4}{9} < 0$.

Substituting $t_2 = t_1$ in (38) leads to:

$$\begin{aligned}W_2^{NR} = {} & \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_1\right)^2 + 2\left(\frac{A-t_1}{3}\right)^2 + t_1\left(\frac{2}{3}(A-t_1)\right) \\ & - (1-\gamma)\,d\left(\frac{4}{3}(A-t_1)\right)\end{aligned} \tag{39}$$

where W_2^{NR} is also concave in t_1.

Finally, in the case $t_1 < t_2$: $W_2^{TR_2}$ reads

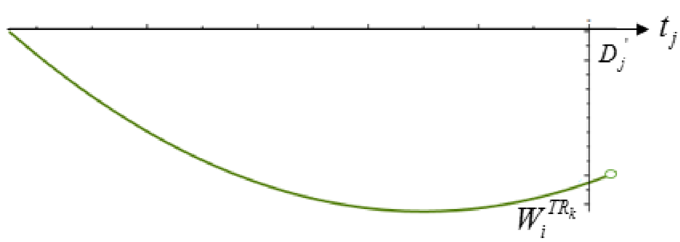
$$W_2^{TR_2} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}t_1\right)^2 - (1-\gamma)\,d\left(\frac{4}{3}(A-t_1)\right) \tag{40}$$

A.4 Proof of Lemma 1

Using Appendix A.2 and A.3: (i) First, it is clear that each country achieves the highest welfare level if it attracts all plants and imposes its unconstrained carbon tax. Therefore, we always have $\hat{W}_i^{TR_\ell} > W_i^{NR}$ and $\hat{W}_i^{TR_\ell} > W_i^{TR_k}$. Second, W_i^{NR} and $W_i^{TR_k}$ intersect at two levels of t_j : $-\frac{1}{2}A$ and A, where $W_i^{NR} > W_i^{TR_k}$ for all $t_j \in \left(-\frac{1}{2}A,\, A\right)$ and $W_i^{NR} < W_i^{TR_k}$ for all $t_j < -\frac{1}{2}A$. Therefore, for $t_j \in \left(D_i',\, A\right)$, we have $\hat{W}_i^{TR_\ell} > W_i^{NR} > W_i^{TR_k}$. (ii) $\tilde{W}_i^{TR_\ell}$ intersects with W_i^{NR} at two levels of t_j which are A and $-\frac{1}{2}A$ for $\varepsilon \to 0$, where $\tilde{W}_i^{TR_\ell} > W_i^{NR}$ for all $t_j \in \left(-\frac{1}{2}A,\, A\right)$ and $W_i^{NR} < \tilde{W}_i^{TR_\ell}$ for all $t_j < -\frac{1}{2}A$. Therefore, for $t_j \in \left(-\frac{1}{2}A,\, D_i'\right]$, we have $\tilde{W}_i^{TR_\ell} > W_i^{NR} > W_i^{TR_k}$. (iii) The three curves intersect at $t_j = -\frac{1}{2}A$ for $\varepsilon \to 0$. Finally, for all $t_j < -\frac{1}{2}A$, we have $W_i^{TR_k} > W_i^{NR} > \tilde{W}_i^{TR_\ell}$.

A.5 Proof of Proposition 3 and Proposition 4

First, we compare the welfare level of each leading country under its two choices. For country 1, we find that $W_1^{*TR_1}\left(t_1^{*L} > t_2^{*F} = (1-\gamma)\,d\right) > W_1^{*NR}\left(t_1^{*L} = t_2^{*F} = -\frac{1}{2}A\right)$ if $A < \hat{A}_1^{\,TR_1} = 2\gamma d + \frac{2}{5}d$. With respect to country 2, we find that $W_2^{*TR_2}\left(t_2^{*L} > t_1^{*F} = \gamma d\right) > W_2^{*NR}\left(t_2^{*L} = t_1^{*F} = -\frac{1}{2}A\right)$ if $A < \hat{A}_2^{\,TR_2} = 2(1-\gamma)d + \frac{2}{5}d$. However, in this case, there is a contradiction with $A > d$ for all $\gamma > 0.7$. In other words, we have $\hat{A}_2^{\,TR_2} > d$ only if $\gamma < 0.7$. Second, we check that if the leader chooses to let its firm to relocate, it always prefers the follower to choose its optimal tax rather than undercutting. Due to the convexity of the function $W_i^{TR_k}$ and the fact that W_i^{NR} and $W_i^{TR_k}$ intersect at $t_j = -\frac{1}{2}A$, as shown in Appendix A.4, if $-\frac{1}{2}A$ lies above the minimum tax level, W_i^{*NR} is strictly lower than $W_i^{TR_k}$, where the latter reaches the highest level at the highest possible level of t_j, which is D_j'. On the other hand, if $-\frac{1}{2}A$ lies below the minimum level, it is also clear that if $W_i^{TR_k}$ is larger than W_i^{*NR}, the highest level is achieved if the follower sets its optimal tax, i.e. D_j', otherwise, the leader is better off under NR.



A.6 Proof of Corollary 1

(i) The SE improves upon the NE outcome only if the Stackelberg leader chooses equilibrium 2, where equilibrium 1 is exactly the same as the NE. In such cases, it is clear that both the follower and the leader are better off. Recall that $\hat{W}_i^{TR_\ell}$ dominates any other welfare level. Under the second SE, firms face a carbon tax D_i', which is obviously larger than subsidy level $-\frac{1}{2}A$. Therefore, the second SE reduces global emissions. (*ii*) Compared to the socially optimal carbon tax, we have $t_C - t^{*NE} = \frac{3}{2}d > 0$ and $t_C > t_2^{*F} = (1-\gamma)d$ if $A < d\,(2\gamma+1)$, which already holds under this SE since $d\,(2\gamma+1) > \hat{A}_1^{TR_1}$. Similarly, if country 2 leads, $t_C > t_1^{*F} = \gamma d$ if $A < d\,(3-2\gamma)$,which is already satisfied under the parameter range that support this SE, i.e. $d\,(3-2\gamma) > \hat{A}_2^{TR_2}$.

B Appendix B

B.1 The welfare functions of country 1 under possible location equilibria:

In the case $t_1 > t_2$, there is partial relocation of firm 1. Both plants supplying country 1 face $t_{k1} = t_1$, while those plants supplying country 2 face $t_{k2} = t_2$. Hence, inserting these tax levels into (3) gives the equilibrium output levels under PR_1 as $x_{k1} = (A - t_1)/3$ and $x_{k2} = (A - t_2)/3$.

First, inserting $\hat{t}_1^{PR_1}(t_2) = \gamma d + \frac{1}{2}t_2$ into (17), we obtain:

$$\begin{aligned}\hat{W}_1^{PR_1} &= \frac{1}{2}\left(\frac{2}{3}A - \frac{1}{3}t_2 - \frac{2}{3}\gamma d\right)^2 + \frac{(2A - 2\gamma d - t_2)^2}{36} \\ &+ \frac{(2\gamma d - t_2)\,(2A - 2\gamma d - t_2)}{12} + \frac{(2\gamma d + t_2)\,(2A - 2\gamma d - t_2)}{12} \\ &- \gamma d\left(\frac{4A - 2\gamma d}{3} - t_2\right)\end{aligned} \tag{41}$$

where $\frac{\partial^2 \hat{W}_1^{PR_1}}{\partial t_2^2} = \frac{1}{6}$, i.e. convex in t_2.

Second, inserting $\ddot{t}_1 = A - \varepsilon$ into (17) gives:

$$\ddot{W}_1^{PR_1} = \frac{(2A - t_2 - \varepsilon)\,\varepsilon}{3} - \gamma d\left(\frac{2\,(A - t_2 + \varepsilon)}{3}\right) \tag{42}$$

which is linear in t_2.

Third, inserting $t_1 = \breve{t}_1 = t_2 + \varepsilon$ into (17) gives:

$$\breve{W}_1^{PR_1} = \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}(t_2+\varepsilon)\right)^2 + \frac{(A-(t_2+\varepsilon))^2}{9} + \varepsilon\left(\frac{A-(t_2+\varepsilon)}{3}\right) \quad (43)$$
$$(t_2+\varepsilon)\left(\frac{A-(t_2+\varepsilon)}{3}\right) - \gamma d\left(\frac{4A-4t_2-2\varepsilon}{3}\right)$$

which is linear in t_2.

B.2 Details of Lemma 2

First, we state the conditions that satisfy the NNC and the BCA constraint mentioned in the text. Recall that we assume throughout the paper that $A > d$ and $t_i < A$. Hence, for country 1 to impose its optimal tax in (18), $\hat{t}_1^{PR_1}$, both constraints need to be satisfied. On the one hand, to satisfy the NNC, we need $\hat{t}_1^{PR_1} < A$, which puts a constraint on t_2 such that $t_2 < 2A - 2\gamma d$. On the other hand, the BCA constraint requires $t_2 < \hat{t}_1^{PR_1}$, which also puts a constraint on t_2 such that $t_2 < 2\gamma d$. We find that if $A > 2\gamma d$, then $2A - 2\gamma d > A$, and since we assume that $t_2 < A$, hence $t_2 < 2A - 2\gamma d$, i.e. the NNC is satisfied. However, the BCA constraint is not necessarily satisfied since we may have $A > t_2 > 2\gamma d$. While if $A < 2\gamma d$, we have $2A - 2\gamma d < A$, which may lead to $2A - 2\gamma d < t_2 < A$, i.e. the NNC is violated. In such cases, the BCA constraint is already satisfied, $t_2 < A < 2\gamma d$. Hence, whenever one of the constraints is violated, we use the welfare function that country 1 imposes a constrained carbon tax as shown in section 4. Second, we compare these two constraints with γd to determine which welfare function to use under TR_2 and PR_1 in each range of t_2. Of course, the welfare function under NR is the same for all cases. Obviously, $2\gamma d > \gamma d$, hence, if $t_2 > 2\gamma d$, t_2 is necessarily larger than γd. This implies that if the BCA constraint is violated, the constrained welfare level under PR_1, i.e. $\breve{W}_1^{PR_1}$, is compared to the unconstrained welfare level under TR_2, i.e. $\hat{W}_1^{TR_2}$. Regarding the NNC, we find that if $\frac{3}{2}\gamma d < A$, then $\gamma d < 2A - 2\gamma d$. As a result, if $t_2 > 2A - 2\gamma d$, i.e. if the NNC is violated, we must have $t_2 > \gamma d$. In such cases, the constrained welfare level under PR_1, $\ddot{W}_1^{PR_1}$ is compared with the unconstrained welfare level $\hat{W}_1^{TR_2}$. Finally, if $A < \frac{3}{2}\gamma d$, we have $2A - 2\gamma d < \gamma d$. This implies that if $2A - 2\gamma d < \gamma d < t_2$, we compare the constrained welfare level $\ddot{W}_1^{PR_1}$ with the unconstrained welfare level $\hat{W}_1^{TR_2}$. While if $2A - 2\gamma d < t_2 < \gamma d$, we compare the constrained welfare level $\ddot{W}_1^{PR_1}$ with the constrained welfare level $\tilde{W}_1^{TR_2}$. We divide Lemma 2 into three cases depending on the range of the parameter values. In the first case, the BCA constraint may be violated, while in the second and the third case, the NNC may be violated.

Lemma. *2(a) If $A > 2\gamma d$:*

i. if $2\gamma d < t_2 < A$, the welfare levels of country 1 can be ranked as: $\hat{W}_1^{TR_2} > W_1^{NR} > \breve{W}_1^{PR_1}$.

ii. if $\gamma d < t_2 < 2\gamma d$ the welfare levels of country 1 can be ranked as: $\hat{W}_1^{TR_2} > W_1^{NR}$ and $\hat{W}_1^{TR_2} > \hat{W}_1^{PR_1}$.

iii. if $\underline{t_2} \leq t_2 < \gamma d$, the welfare levels of country 1 can be ranked as: $\tilde{W}_1^{TR_2} > W_1^{NR}$ and $\tilde{W}_1^{TR_2} \geq \hat{W}_1^{PR_1}$.

iv. if $-\frac{1}{2}A < t_2 < \underline{t_2}$, the welfare levels of country 1 can be ranked as: $\hat{W}_1^{PR_1} > \tilde{W}_1^{TR_2} > W_1^{NR}$.

v. if $t_2 \leq -\frac{1}{2}A$, the welfare levels of country 1 can be ranked as: $\hat{W}_1^{PR_1} > W_1^{NR} \geq \tilde{W}_1^{TR_2}$.

Proof. Using Appendix A.2 and B.1, *(i)* As mentioned in Appendix A.4, it is clear that as long as $\hat{W}_1^{TR_2}$ is feasible, it dominates all other welfare levels. In this case, we always have $\hat{W}_1^{TR_2} > \breve{W}_1^{PR_1}$ and $\hat{W}_1^{TR_2} > W_1^{NR}$. In addition, $\breve{W}_1^{PR_1}$ and W_1^{NR} intersect at two tax levels, which are A and $-\frac{1}{2}A$ for all $\varepsilon \to 0$ such that $W_1^{NR} < \breve{W}_1^{PR_1}$for all $t_2 > A$, which is not feasible, and for all $t_2 < -\frac{1}{2}A$, which is not included in this range. Therefore, for $2\gamma d < t_2 < A$, we have $\hat{W}_1^{TR_2} > W_1^{NR} > \breve{W}_1^{PR_1}$. *(ii)* Since $\hat{W}_1^{TR_2}$ is till feasible, we have $\hat{W}_1^{TR_2} > \hat{W}_1^{PR_1}$ and $\hat{W}_1^{TR_2} > W_1^{NR}$. *(iii)* $\tilde{W}_1^{TR_2}$ intersects with$\hat{W}_1^{PR_1}$ at two tax levels of $t_2 :\frac{2}{9}\left(A+\gamma d \pm \Theta\right)$ for all $\varepsilon \to 0$, where $\Theta = \sqrt{2\left(A+\gamma d\right)\left(5A-4\gamma d\right)}$, such that $\hat{W}_1^{PR_1} > \tilde{W}_1^{TR_2}$ for all $t_2 \in \left(\frac{2}{9}\left(A+\gamma d+\Theta\right), \infty\right)$ and for all $t_2 \in \left(-\infty, \frac{2}{9}\left(A+\gamma d-\Theta\right)\right)$. The larger tax level is higher than γd for all $A > 1.08\gamma d$, so it is not included in this range since we have $A > 2\gamma d$. The smaller value of t_2 is less than zero for all $A > \gamma d$, which holds since we assume $A > d$, and is also larger than $-\frac{1}{2}A$. We denote this tax level by $\underline{t_2}\left(\gamma, d, A\right)$ in the text, where $\underline{t_2} =\frac{2}{9}\left(A+\gamma d-\Theta\right)$. Therefore, we have $\tilde{W}_1^{TR_2} \geq \hat{W}_1^{PR_1}$ for all $t_2 \geq \underline{t_2}$. From Appendix A.4, we have $\tilde{W}_1^{TR_2} \leq W_1^{NR}$ for all $t_1 \leq -\frac{1}{2}A$. Therefore, in this range we have $\tilde{W}_1^{TR_2} > W_1^{NR}$. In (iv), $\hat{W}_1^{PR_1}$ intersects with W_1^{NR} at two levels of t_2: $\frac{2}{11}(A+3\gamma d) \pm \psi$, where $\psi = \frac{2}{11}\sqrt{6(2A^2 + A\gamma d - 3\gamma^2 d^2)}$, such that $W_1^{NR} > \hat{W}_1^{PR_1}$ for all $t_2 \in \left(\frac{2}{11}(A+3\gamma d) \pm \psi\right)$. However, these tax values are not included in this range, where the larger value is higher than γd for all $A > 2\gamma d$, while the smaller value is larger than $-\frac{1}{2}A$, and also larger than $\underline{t_2}$ for all $A > 2\gamma d$. Hence, in (iv), we have $\hat{W}_1^{PR_1} > \tilde{W}_1^{TR_2} > W_1^{NR}$. Finally, for all $t_1 \leq -\frac{1}{2}A$, we have $\hat{W}_1^{PR_1} > W_1^{NR} \geq \tilde{W}_1^{TR_2}$. □

Lemma. *2(b) If $\frac{3}{2}\gamma d < A < 2\gamma d$: this implies that $\gamma d < 2A - 2\gamma d < A$*

i. if $2A - 2\gamma d < t_2 < A$, the welfare levels of country 1 can be ranked as: $\hat{W}_1^{TR_2} > W_1^{NR} > \ddot{W}_1^{PR_1}$.

ii. if $\gamma d < t_2 < 2A - 2\gamma d$, the welfare levels of country 1 can be ranked as: $\hat{W}_1^{TR_2} > W_1^{NR}$ and $\hat{W}_1^{TR_2} > \hat{W}_1^{PR_1}$.

iii. if $\underline{t_2} < t_2 < \gamma d$, the welfare levels of country 1 can be ranked as: $\tilde{W}_1^{TR_2} > W_1^{NR}$ and $\tilde{W}_1^{TR_2} > \hat{W}_1^{PR_1}$.

iv. if $-\frac{1}{2}A < t_2 < \underline{t_2}$*, the welfare levels of country 1 can be ranked as:* $\hat{W}_1^{PR_1} > \tilde{W}_1^{TR_2} > W_1^{NR}$.

v. if $t_2 \leq -\frac{1}{2}A$*, the welfare levels of country 1 can be ranked as:* $\hat{W}_1^{PR_1} > W_1^{NR} \geq \tilde{W}_1^{TR_2}$.

Proof. (i) W_1^{NR} intersects with $\ddot{W}_1^{PR_1}$ at two levels of t_2, which are A and $-2A + 3\gamma d$ for $\varepsilon \to 0$, such that $\ddot{W}_1^{PR_1} > W_1^{NR}$ for all $t_2 \in (A, \infty)$ and for all $t_2 \in (-\infty, -2A + 3\gamma d)$. The first solution is not feasible since it is larger than A, while the second solution is less than $2A - 2\gamma d$, hence is not included in this range. Therefore, we have $\hat{W}_1^{TR_2} > W_1^{NR} > \ddot{W}_1^{PR_1}$. From *(ii)* to *(v)* the BCA constraint and the NNC are satisfied, hence we have the same proof as in the previous sub-case. □

Lemma. *2(c) If* $\gamma d \leq d < A < \frac{3}{2}\gamma d$*, this means that* $2A - 2\gamma d < \gamma d < A$

i. if $\gamma d < t_2 < A$*, the welfare levels of country 1 can be ranked as:* $\hat{W}_1^{TR_2} > W_1^{NR} > \ddot{W}_1^{PR_1}$.

ii. if $2A - 2\gamma d < t_2 < \gamma d$*, the welfare levels of country 1 can be ranked as:* $\tilde{W}_1^{TR_2} > W_1^{NR} > \ddot{W}_1^{PR_1}$.

iii. if $\underline{t_2} < t_2 < 2A - 2\gamma d$*, the welfare levels of country 1 can be ranked as:* $\tilde{W}_1^{TR_2} > W_1^{NR}$ *and* $\tilde{W}_1^{TR_2} > \hat{W}_1^{PR_1}$.

iv. if $-\frac{1}{2}A < t_2 \leq \underline{t_2}$*, the welfare levels of country 1 can be ranked as:* $\hat{W}_1^{PR_1} \geq \tilde{W}_1^{TR_2} > W_1^{NR}$.

v. if $t_2 \leq -\frac{1}{2}A$*, the welfare levels of country 1 can be ranked as:* $\hat{W}_1^{PR_1} > W_1^{NR} \geq \tilde{W}_1^{TR_2}$.

Proof. (i) In this range, the NNC is also violated under PR_1, hence, we have the same proof as for the first range in the previous sub-case. *(ii)* For this range, $\tilde{W}_1^{TR_2}$ intersects with $\ddot{W}_1^{PR_1}$ at two levels of t_2 : A and $\gamma d - A$ for $\varepsilon \to 0$, such that $\ddot{W}_1^{PR_1} > \tilde{W}_1^{TR_2}$ for all $t_2 \in (A, \infty)$ and for all $t_2 \in (-\infty, \gamma d - A)$. However, the first solution is not feasible. In addition, we have $\gamma d - A < 0$ for all $A > \gamma d$, which holds since we have $A > d$. On the other hand, this range assumes $t_2 > 2A - 2\gamma d$, where $2A - 2\gamma d \geq 0$ for all $A \geq \gamma d$. Hence, the second solution is not included in this range. Therefore, we have $\tilde{W}_1^{TR_2} > W_1^{NR} > \ddot{W}_1^{PR_1}$. Recall that we also have $\tilde{W}_1^{TR_2} > W_1^{NR}$ from Appendix A.4. From *(iii)* to *(v)* we have the same proof as in the previous sub-cases. □

B.3 The welfare functions of country 2 under possible location equilibria:

In the case of $t_1 > t_2$, first: substituting $t_2 = \hat{t}_2^{PR_1}(t_1) = \frac{1}{4}A - \frac{3}{4}t_1 + \frac{3}{2}d\,(1-\gamma)$ into (19),we obtain:

$$
\begin{aligned}
\hat{W}_2^{PR_1} &= \frac{1}{2}\left(\frac{1}{2}A + \frac{1}{2}t_1 - (1-\gamma)d\right)^2 + \frac{(A + t_1 - (2(1-\gamma)d))^2}{8} \\
&+ \frac{(A-t_1)^2}{9} + \frac{(A - 3t_1 + 6\,(1-\gamma)\,d)\,(5A + t_1 - 6\,(1-\gamma)\,d)}{24} \\
&- (1-\gamma)\,d\left(\frac{7}{6}A - \frac{1}{6}t_1 - (1-\gamma)d\right) \qquad (44)
\end{aligned}
$$

where $\frac{\partial^2 \hat{W}_2^{PR_1}}{\partial t_1^2} = \frac{17}{36}$, i.e. convex in t_1.

Second, inserting $t_2 = t_1 - \varepsilon$ into (19) gives:

$$
\begin{aligned}
\tilde{W}_2^{PR_1} &= \frac{1}{2}\left(\frac{2}{3}A - \frac{2}{3}\,(t_1 - \varepsilon)\right)^2 + 2\left(\frac{(A - (t_1 - \varepsilon))^2}{9}\right) + \frac{(A-t_1)^2}{9} \\
&+ (t_1 - \varepsilon)\left(A - t_1 + \frac{2}{3}\varepsilon\right) - (1-\gamma)\,d\left(\frac{4A - 4t_1 + 2\varepsilon}{3}\right) \qquad (45)
\end{aligned}
$$

where $\frac{\partial^2 \tilde{W}_2^{PR_1}}{\partial t_1^2} = -\frac{8}{9}$, i.e. is concave in t_1.

B.4 Proof of Lemma 3.

To satisfy the NNC, we need $\hat{t}_2^{PR_1}(t_1) < A$, which requires $t_1 > 2\,(1-\gamma)\,d - A$. With respect to the BCA constraint, it requires $t_1 > \hat{t}_2^{PR_1}(t_1)$. For this to be true, we must have $t_1 > \bar{t}_1$. We find that $\bar{t}_1 > 2\,(1-\gamma)\,d - A$, for all $A > (1-\gamma)\,d$, which generally holds. Thus, we only have to consider the more strict condition.

Using Appendix A.3 and B.3, (i) for the range where $t_1 > \bar{t}_1$, country 2 sets $\hat{t}_2^{PR_1}(t_1)$. For this range, $\hat{W}_2^{PR_1}$ and W_2^{NR} intersect at two levels of $t_1:\ \frac{1}{11}\,(A + 14\,(1-\gamma)\,d) \pm \Omega$, where $\Omega = \frac{4}{11}\sqrt{4\,(\gamma-1)^2\,d^2 + A\,(\gamma-1)\,d - 2A^2}$, such that $W_2^{NR} > \hat{W}_2^{PR_1}$ for all $t_2 \in \left(\frac{1}{11}\,(A + 14\,(1-\gamma)\,d) \pm \Omega\right)$. However, these two tax levels are larger than A, for all $0 < A < (1-\gamma)\,d$, i.e this solution is not feasible. In addition, for all $A > d$, these two tax levels are not defined. Hence, we always have $\hat{W}_2^{PR_1} > W_2^{NR}$. From Appendix A.4, we have $W_2^{NR} < W_2^{TR_2}$ for all $t_1 < -\frac{1}{2}A$. In addition, we have $\bar{t}_1 > -\frac{1}{2}A$ for all $A > -\frac{4}{3}d\,(1-\gamma)$, which obviously holds since $A > 0$. Therefore, we have $\hat{W}_2^{PR_1} > W_2^{NR} > W_2^{TR_2}$. (ii) If $t_1 < \bar{t}_1$, country 2 sets its constrained carbon tax, i.e. $t_1 - \varepsilon$, where $\tilde{W}_2^{PR_1}$ intersects with W_2^{NR} at two tax levels of t_1, which are again A and $-\frac{1}{2}A$ for $\varepsilon \to 0$, where $\tilde{W}_2^{PR_1} < W_2^{NR}$ for all $t_1 < -\frac{1}{2}A$. Therefore, for this range we have $\tilde{W}_2^{PR_1} > W_2^{NR} > W_2^{TR_2}$. The three curves intersect at $t_1 = -\frac{1}{2}A$ for $\varepsilon \to 0$. (iii) Finally, for all $t_1 \leq -\frac{1}{2}A$, we have $W_2^{TR_2} \geq W_2^{NR} \geq \tilde{W}_2^{PR_1}$.

B.5 Proof of Proposition 5.

(i) The proof follows directly from Fig. 8 (ii) A NE exists if $\underline{t_2} \geq t_2^{*NE}(PR_1)$, where $t_2^{*NE}(PR_1)$ is derived in (24) and $\underline{t_2} = \frac{2}{9}(A + \gamma d - \Theta)$ as defined in Appendix B.2. This inequality holds if and only if $A \leq \overline{A}_{NE} = \frac{d\left(7\gamma - 12 + 33\sqrt{129\gamma^2 - 136\gamma + 40}\right)}{134} + \varepsilon$. This threshold of market size increases in d, where the term in brackets is positive for all values of γ. In addition, this term increases in γ for all $\gamma > 0.52$. However, this threshold level is not always feasible. In other words, we have $\overline{A}_{NE} > d$ for all $\gamma > 0.855$ only. Hence, existence of a NE requires d and γ to be large. (iii) If a NE exits, it is clear from (23) that the climate policy of country 1 is a tax. With respect to country 2, the equilibrium climate policy is a subsidy. Since a NE exists only if $\underline{t_2} \geq t_2^{*NE}(PR_1)$, and, as mentioned in Appendix B.2, $\underline{t_2} < 0$, hence, $t_2^{*NE}(PR_1)$ is a subsidy.

B.6 Proof of Proposition 6.

Country 1 chooses between PR_1 and NR. $W_1^{*L}(NR)$ is the same as in Appendix A.5. By inserting $t_1^{*L}(PR_1)$ and $t_2^{*F}(PR_1)$ derived in (25) and (26) into (17), we obtain $W_1^{*L}(PR_1)$. We find that $W_1^{*L}(PR_1) > W_1^{*L}(NR)$ for all $A < \hat{A}_1^{PR_1} = \frac{d\left(\sqrt{42}\sqrt{32\gamma^2 - 8\gamma + 9} + 52\gamma - 15\right)}{17}$, where this threshold of market size increases in d and γ since the term in brackets is larger than zero and increases in γ for all levels of γ. In addition, this threshold level is always feasible, i.e. $\hat{A}_1^{PR_1} > d$. To check that country 1 does not prefer to set $-\frac{1}{2}A < t_1 < \bar{t}_1$, such that country 2 responds by undercutting t_1, we insert $t_2 = t_1 - \varepsilon$ into (17)and obtain $\widetilde{W_1}^L(PR_1)$. We find that $W_1^{*L}(PR_1) = \widetilde{W_1}^L(PR_1)$ at $\widetilde{t}_1 = \frac{3A^2 + (30 - 48\gamma)Ad + \left(80\gamma^2 - 72\gamma - 9\right)d^2}{28(A - 4\gamma d)}$ for $\varepsilon \to 0$. On the one hand, $W_1^{*L}(PR_1) < \widetilde{W_1}^L(PR_1)$ for all $t_1 \in \left(\widetilde{t}_1, \infty\right)$ if $A < 4\gamma d$, which implies that $A < \hat{A}_1^{PR_1}$. In this case, $\widetilde{t}_1$ is a positive value, nevertheless, we have $\widetilde{t}_1 > \bar{t}_1$. Hence, country 2 would react on its standard reaction function, where country 1 uses $t_1^{*L}(PR_1)$. On the other hand, we find that $W_1^{*L}(PR_1) < \widetilde{W_1}^L(PR_1)$ for all $t_1 \in \left(-\infty, \widetilde{t}_1\right)$ if $A > 4\gamma d$, where $\widetilde{t}_1$ becomes a negative value. However, if $A > 4\gamma d$, but $A \leq \hat{A}_1^{PR_1}$, we find that $\widetilde{t}_1 \leq -\frac{1}{2}A$, hence undercutting is not a best response for country 2. Finally, if $A > \hat{A}_1^{PR_1}$, which implies $A > 4\gamma d$, we have $W_1^{*L}(PR_1) < \widetilde{W_1}^L(PR_1)$ for all $t_1 \in \left(-\infty, \widetilde{t}_1\right)$, where $\widetilde{t}_1 > -\frac{1}{2}A$. However, in this case $W_1^{*L}(NR) > \widetilde{W_1}^L(PR_1)$ for all $t_1 > -\frac{1}{2}A$. Therefore, to sum up, for all $A < \hat{A}_1^{PR_1}$, we have $W_1^{*L}(PR_1) > W_1^{*L}(NR)$ and $W_1^{*L}(PR_1) > \widetilde{W_1}^L(PR_1)$. Whereas, for all $A > \hat{A}_1^{PR_1}$, we have $W_1^{*L}(NR) > W_1^{*L}(PR_1)$ and $W_1^{*L}(NR) > \widetilde{W_1}^L(PR_1)$.

B.7 Proof of Proposition 7

Country 2 chooses between PR_1 and TR_2. By inserting (27)and (28) into (19), we obtain $W_2^{*L}(PR_1)$, which is feasible only for all $\gamma > \widehat{\gamma}$ if $A < \overline{A}_{SE}$. However, if these conditions do not hold, country 2 sets its tax marginally below $\underline{t_2}$ and country 1 sets its tax at the level in (18). Inserting these tax levels into (19), we obtain $\underline{W_2^{*L}}(PR_1)$. These equilibria under PR_1 need to be compared with the welfare level under TR_2. There are two best responses for country 1: $t_1 = \gamma d$ and $t_1 = t_2 - \varepsilon$. Inserting these tax levels into (40), we obtain $W_2^{*L}(TR_2)$ and $\widetilde{W_2}^{L}(TR_2)$, respectively. First, we find that $W_2^{*L}(TR_2) > W_2^{*L}(PR_1)$ if $A < \frac{d\left(\sqrt{10764\gamma - 7085\gamma^2 - 3159} - 13\gamma\right)}{39}$, which violates our assumption that $A > d$. Due to the complexity of the formula $\underline{t_2}$, sometimes it is not possible to obtain analytical solutions, thus we do numerical simulations. We find that $\widetilde{W_2}^{L}(TR_2) \geq W_2^{*L}(PR_1)$ if $t_2 \in \left(-\infty, \underset{\sim}{t_2}\right]$, where $\underset{\sim}{t_2} = A - 3d\,(1-\gamma) - \frac{1}{26}\sqrt{(7540\gamma^2 - 15288\gamma + 8190)\,d^2 + (3380\gamma - 4056)\,Ad + 1690A^2}$. However, we checked that $\underline{t_2} > \underset{\sim}{t_2}$ for all $A > d$. Hence, since $\underline{t_2} \notin \left(-\infty, \underset{\sim}{t_2}\right]$, country 1 will not respond by undercutting. Therefore, we always have $W_2^{*L}(PR_1) > W_2^{*L}(TR_2)$ and $W_2^{*L}(PR_1) > \widetilde{W_2}^{L}(TR_2)$. Second, we find that $\underline{W_2^{*L}}(PR_1) > W_2^{*L}(TR_2)$ for all $A > d$. While $\underline{W_2^{*L}}(PR_1) < \widetilde{W_2}^{L}(TR_2)$ if $t_2 \in \left(-\infty, \widetilde{t_2}\right]$, where
$\widetilde{t_2} = A - 3d\,(1-\gamma) - \frac{1}{9}\sqrt{13\sqrt{2}\left(\left(\frac{112\gamma - 81}{13}\right)d + A\right)\sqrt{A + \gamma d}\sqrt{5A - 4\gamma d} + \varpi + \varphi}$
and $\varpi = (729 + 473\gamma^2 - 1134\gamma)\,d^2$ and $\varphi = (280\gamma - 405)\,Ad + 131A^2$. However, country 1 responds by undercutting only if $t_2 > \underline{t_2}$. We find that $\widetilde{t_2} > \underline{t_2}$ if and only if $A > \overline{A}^{TR_2} = d\left(6 + \frac{1}{2}\left(3\sqrt{\gamma^2 - 8\gamma + 10} - 5\gamma\right)\right)$, where the term in brackets is larger than 1, hence this threshold increases in d, but decreases in γ. Therefore, if this condition holds, i.e. if $\underline{t_2} \in \left(-\infty, \widetilde{t_2}\right]$, the lowest possible tax that country 2 can set to induce country 1 to undercut its tax is marginally above $\underline{t_2}$.

B.8 Proof of Corollary 2

i. In the simultaneous game, we have $t_1^{*NE}(PR_1) > t_C$ if $A > \frac{d(21-4\gamma)}{13}$ $\forall\, \gamma > 0.9765$, where this threshold level increases in A and decreases in γ. While $t_2^{*NE}(PR_1) > t_C$ if $A > \frac{3d(1+4\gamma)}{5}$, which is larger than $\overline{A}_{NE}$. Hence, for the parameter values that supports the existence of a NE we have $t_2^{*NE}(PR_1) < t_C$. Global emission level is higher under the social optimum, $E_C > E^{*NE}(PR_1)$, if $A > \frac{d(15+16\gamma)}{14}$. However, a NE does not exists if this condition holds. Hence, we have $E_C < E^{*}(PR_1)$. *ii*. (*a*) If country 1 leads and chooses BCAs, we have $t_1^{*L}(PR_1) > t_C$ if $A > \frac{d(15+4\gamma)}{11}$, which increases in both γ and d. In addition, $t_2^{*F}(PR_1) > t_C$ if $A > \frac{3d(1+4\gamma)}{5}$, which similarly increases in γ and d. Global emissions are larger under the social optimum if $A > \frac{d(39+44\gamma)}{37}$. (*b*) If country 2 leads and chooses equilibrium 1 stated in Proposition

7: first, we have $t_1^{*F}(PR_1) > t_C$ if $A > \frac{d(21-4\gamma)}{13}$ $\forall\,\gamma > 0.9765$, while $t_2^{*L}(PR_1) > t_C$ if $A > \frac{d(3+44\gamma)}{13}$, which is larger than $\overline{A}_{SE}$. Thus, we have $t_2^{*L}(PR_1) < t_C$. Global emissions are larger under the social optimum if $A > \frac{4d(3+5\gamma)}{13}$, which is also larger than $\overline{A}_{SE}$. Hence, global emissions are less in the social optimum solution. Second, we have $t_C < t_2^{*L}(PR_1) \lesssim \underline{t_2}$ if $A > 4d\left(3\sqrt{2}\sqrt{5-\gamma} - \gamma + \frac{39}{4}\right)$, which is larger than $A > \overline{A}^{TR_2}$. Hence, we have $\underline{t_2} < t_C$. While $\hat{t}_1^{PR_1}\left(\underline{t_2}\right) = t_1^{*F} > t_C$ if $A > \frac{d\left(2\sqrt{2}\sqrt{2\gamma^2-7\gamma+5}-8\gamma+11\right)}{3}$, which increases in d and decreases in γ. Global emissions are higher under the social optimum if $A > \frac{d\left(3\sqrt{2}\sqrt{2\gamma^2-16\gamma+20}-10\gamma+24\right)}{4}$, which also increases in d and decreases in γ. If country 2 chooses equilibrium 2, we have $t_2^{*L} \gtrsim t_1^{*F} \gtrsim \underline{t_2}$. We have shown above that $\underline{t_2} < t_C$, thus, global emissions are lower under the social optimum.

C Appendix C

C.1 Proof of Corollary 4.

Inserting the equilibrium tax levels in (23) and (24) into (17) and (19), we obtain $W_1^{*PR_1}$ and $W_2^{*PR_1}$. Similarly, by inserting $t_i^{*NE} = -\frac{1}{2}A$ into (32) and (38), we obtain W_1^{*NR} and W_2^{*NR}. (i) $W_1^{*NR} > W_1^{*PR_1}$ if $\beta \in \left(-\infty, \frac{488\gamma-120-\xi}{163}\right)$or if $\beta \in \left(\frac{488\gamma-120+\xi}{163}, \infty\right)$, where $\xi = 22\sqrt{6}\sqrt{38\gamma^2-8\gamma+9}$. The first solution violates the NNC, while the second solution violates the threshold level under which the NE exists. Hence, we have $W_1^{*NR} < W_1^{*PR_1}$. (ii) $W_2^{*NR} < W_2^{*PR_1}$ $\forall\gamma < 0.96$ and $\forall\, 0.96 < \gamma < 0.977$ if $\beta < \frac{976\gamma-768-132\sqrt{49\gamma^2-74\gamma+26}}{95}$, which decreases in γ. While, $W_2^{*NR} > W_2^{*PR_1}$ $\forall\,\gamma > 0.977$.(iii) Global welfare is larger under NR if $\beta > \frac{204+244\gamma+66\sqrt{12\gamma-16\gamma^2+47}}{197}$. However, under this condition a NE does not exist. Therefore, as long as $A < \overline{A}_{NE}$, global welfare level is higher under PR_1, i.e. with BCAs.

C.2 Proof of Corollary 5.

From proposition 3 and 6, we find that $\hat{A}_1^{TR_1} < \hat{A}_1^{PR_1}$. This allows us to divide Fig. 9a into three regions: Region F, where $A < \hat{A}_1^{TR_1}$, region G, where $\hat{A}_1^{TR_1} < A < \hat{A}_1^{PR_1}$ and region H, where $\hat{A}_1^{TR_1} < \hat{A}_1^{PR_1} < A$. In region F, we compare (25) and (26) with $(1-\gamma)\,d$. We find that $t_1^{*L}(PR_1) > (1-\gamma)\,d$ for all values of γ as long as $A > d$. On the other hand, $t_2^{*F}(PR_1) > (1-\gamma)\,d$ for all $\gamma < 0.75$ and if $\beta > 8\gamma - 5$ for all $0.75 < \gamma < 0.9$, while $t_2^{*F}(PR_1) < (1-\gamma)\,d$ for all $\gamma \geq 0.9$. Global emissions are higher with BCAs if $\beta < \frac{11-12\gamma}{9}$, which violates the NNC. Hence, global emissions decrease with BCAs. Inserting $t_2 = (1-\gamma)\,d$ into (34), we obtain $W_1^{*L}(TR_1)$. In addition, from Appendix B.6, we have $W_1^{*L}(PR_1)$. We find that

$W_1^{*L}(TR_1) > W_1^{*L}(PR_1)$ if $\beta < \frac{2\sqrt{126\gamma(1-\gamma)+168}-16\gamma-11}{19}$, which violates the NNC for all values of γ. Hence, we have $W_1^{*L}(TR_1) < W_1^{*L}(PR_1)$. On the other hand, by comparing $W_2^{*F}(PR_1)$, which is derived in Appendix B.7, to (36), it is obvious that country 2 becomes worse off with BCAs. Global welfare level is higher under the BCA-regime if $\beta < \underline{\beta}^{WF} = \frac{403-68\gamma+28\sqrt{90\gamma^2+108\gamma-31}}{317}$, which increases with γ. Since $A < \hat{A}_1^{TR_1}$, this threshold level is satisfied for all $\gamma < 0.83$. However, if $\gamma > 0.83$, we need $\beta < \underline{\beta}^{WF}$ to hold. In region G, the tax levels in (25) and (26) are larger than $-\frac{1}{2}A$ since $A > d$. Thus global emissions decrease with BCAs. It is straightforward that country 1 is better off as long as it chooses PR_1. Regarding country 2, we insert $t_2 = -\frac{1}{2}A$ into (38) which delivers $W_2^{*F}(NR)$. We find that $W_2^{*F}(NR) > W_2^{*F}(PR_1)$ if $\beta < \frac{1468\gamma+42\sqrt{1216\gamma^2-2360\gamma+1145}}{5} - 285$. This threshold level violates the NNC for all $\gamma < 0.98$, thus $W_2^{*F}(NR) < W_2^{*F}(PR_1)$. However, if $\gamma > 0.98$, this threshold level is satisfied since $A < \hat{A}_1^{PR_1}$, hence $W_2^{*F}(NR) > W_2^{*F}(PR_1)$. Global welfare increases under the BCA-regime if $\beta < \underline{\beta}^{WG} = \frac{795+716\gamma+42\sqrt{192\gamma-256\gamma^2+1382}}{709}$, which increases in γ. Similarly, we find that this threshold level is satisfied in this region if $\gamma > 0.98$. However, if $\gamma > 0.98$, we need $\beta < \underline{\beta}^{WG}$ to hold. In region H, we have the same outcome with and without BCAs.

C.3 Proof of Corollary 6.

From proposition 4 and 7, we have $\hat{A}_2^{TR_2} < \overline{A}^{TR_2}$. Therefore, we divide Fig.9b into three regions of parameter values. Region M, where $A < \hat{A}_2^{TR_2}$ for all $\gamma < 0.7$, region N, where $\hat{A}_2^{TR_2} < A < \overline{A}^{TR_2}$ and region O, where $\overline{A}^{TR_2} < A$. First, in region M, all firms are subject to tax equal to γd under the No-BCA-regime. With BCAs, country 2 imposes $t_2^{*L} \lesssim \underline{t_2}$, where $\underline{t_2} < 0$ as shown in Appendix B.2. In addition, from (18), we have $\hat{t}_1^{PR_1} < \gamma d$ since $t_2 < 0$. Hence, both countries impose a lower carbon tax. As a result, global emissions are higher under the BCA-regime. Without BCAs, country 1 achieves the highest welfare level, i.e. $\hat{W}_1^{TR_2}$. Hence, it obviously becomes worse off under the BCA-regime. On the other hand, we have $\underline{W_2^{*L}}(PR_1) > W_2^{*L}(TR_2)$ as mentioned in Appendix B.7. Recall that due to the complexity of $\underline{t_2}$, we sometimes resort to numerical simulations in this Appendix. Global welfare is higher under the No-BCA-regime in this region, i.e. for all $d < A < \hat{A}_2^{TR_2}$ and $\gamma < 0.7$. In region N, firms move from NR to PR_1. In this case, both countries set higher carbon taxes since $\underline{t_2} > -\frac{1}{2}A$. Hence, global emissions are lower under the BCA-regime. For country 1: $W_1^{*F}(PR_1) < W_1^{*F}(NR)$ if $\beta < (>) \frac{48\gamma-18-(+)\sqrt{2}\sqrt{548\gamma^2-360\gamma+243}}{13}$, where the first solution violates the NNC, and the second is larger than $\overline{A}_{SE}$. Hence, $W_1^{*F}(PR_1) > W_1^{*F}(NR)$. Similarly, we find that $\underline{W_1^{*F}}(PR_1) > W_1^{*F}(NR)$ as long as $A < \overline{A}^{TR_2}$. For country 2: $W_2^{*L}(NR) < W_2^{*L}(PR_1)$ if $\gamma < 0.96$ and if $\beta < 8\gamma - 6 - \frac{3\sqrt{26}\sqrt{40\gamma^2-56\gamma+17}}{13}$ for all $0.96 < \gamma < 0.98$. However, this threshold violates the NNC for all $\gamma > 0.98$, where we have $W_2^{*L}(NR) > W_2^{*L}(PR_1)$. Similarly, $W_2^{*L}(NR) < \underline{W_2^{*L}}(PR_1)$ if β is not too large for all $\gamma < 0.965$. However, for all $\gamma \geq 0.965$, $W_2^{*L}(NR) > \underline{W_2^{*L}}(PR_1)$. Global welfare increases in this region since $A < \overline{A}^{TR_2}$. In region O, the equilibrium carbon tax under the BCA-regime is $t_2^{*L} \gtrsim t_1^{*F} \gtrsim \underline{t_2}$, which is larger than $-\frac{1}{2}A$ under the No-BCA-regime. Hence, global emissions are reduced with BCAs. Inserting $t_1 = t_1^{*F} \gtrsim \underline{t_2}$ into both (29) and (40), we obtain $W_1^{*F}(TR_2)$ and $\underline{W_2^{*L}}(TR_2)$, respectively. We find that $W_1^{*F}(TR_2) > W_1^{*F}(NR)$ and $\underline{W_2^{*L}}(TR_2) < W_2^{*L}(NR)$ for all the parameter values in this region. Global welfare is larger under the BCA-regime, i.e. under TR_2 than under NR if $\beta < \underline{\beta}^{WO} = 78 - 4\gamma + 24\sqrt{10-\gamma}$, which increases in d while decreases in γ.

D Appendix D

The case under which the two countries could impose BCAs if $t_i > t_j$.

Based on subsection 2.3, the three possible location equilibria are:NR, PR_1 and PR_2. The first two equilibria are covered in the main text. Under PR_2, i.e. if $t_2 > t_1$, only the plant of firm 2 which supplies the home market remains in country 2, while the other plant that supplies country 1 will relocate.

The welfare functions of country 1 and country 2, respectively, under the three possible location equilibria are given by:

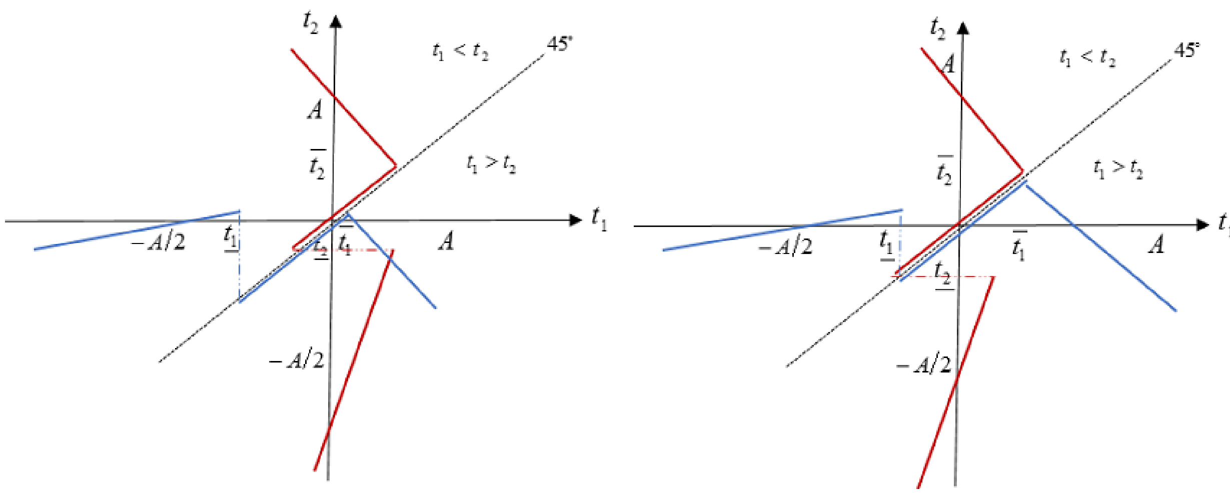
$$W_1 = \begin{cases} W_1^{PR_2} = CS_1 + \pi_{11} + \pi_{12} + \pi_{21} + T_1 - D_1 & \text{if } t_1 < t_2 \\ W_1^{NR} = CS_1 + \pi_{11} + \pi_{12} + T_1 - D_1 & \text{if } t_1 = t_2 \\ W_1^{PR_1} = CS_1 + \pi_{11} + T_1 + BCA_1 - D_1 & \text{if } t_1 > t_2 \end{cases}$$

where $BCA_1 = (t_1 - t_2)\, x_{21}$.

$$W_2 = \begin{cases} W_2^{PR_2} = CS_2 + \pi_{22} + T_2 + BCA_2 - D_2 & \text{if } t_1 < t_2 \\ W_2^{NR} = CS_2 + \pi_{21} + \pi_{22} + T_2 - D_2 & \text{if } t_1 = t_2 \\ W_2^{PR_1} = CS_2 + \pi_{12} + \pi_{21} + \pi_{22} + T_2 - D_2 & \text{if } t_1 > t_2 \end{cases}$$

where $BCA_2 = (t_2 - t_1)\, x_{12}$.

All the analysis that we have done for country 1 under PR_1 will be the the same for country 2 if $t_2 > t_1$. Therefore, also the reaction function of country 2 is discontinuous in this case. Hence, in the simultaneous game, a NE does not always exist as shown in Fig. 10 below.



(a) Existence of a NE 'asymmetric countries'

(b) Non-existence of a NE 'symmetric countries'

Fig. 10: Nash Equilibrium with BCAs by two countries

In case of symmetric countries, no NE exists, as we have shown in the main text. If a NE exists, it would be partial relocation of firm 1, i.e. $t_1 > t_2$. The intersection of the standard reaction functions under PR_2 is not feasible. That is, the condition for $\underline{t_1} > t_1^{*NE}(PR_2)$ is not feasible, where the critical tax level for country 2 is $\underline{t_1} = \frac{2}{19}\left(2A + 3(1-\gamma)d - \sqrt{12(1-\gamma)d(A-4) + 42A^2}\right)$. In addition, we have $t_1^{*NE}(PR_2) > t_2^{*NE}(PR_2)$ for all $\gamma > 0.66$. Therefore, in equilibrium, only country 1 will impose BCAs. Existence of a NE in this case requires the same conditions as shown in section 4, that is, global marginal damages and asymmetry among countries need to be sufficiently high, i.e. if $\underline{t_2} > t_2^{*NE}(PR_1)$ as shown in the paper. However, in this Appendix, $\underline{t_2} = \frac{2}{19}\left(2A + 3\gamma d - \sqrt{12\gamma d(A-4) + 42A^2}\right) + \varepsilon$, which is larger than critical tax level if only country 1 imposes BCAs. Therefore, in this case, a NE is more likely to exist under larger range of values of parameters compared to the case in the main text. It is obvious here that $\underline{t_1} \leq \underline{t_2}$ if $\gamma \geq 0.5$.